MATHEMATICS 10

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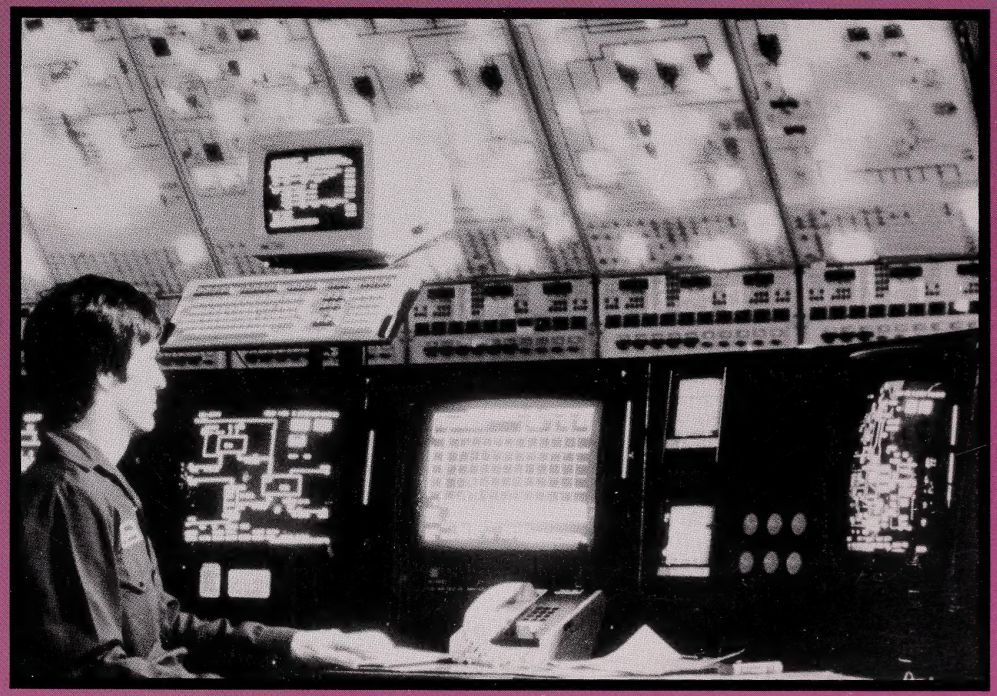
UNIT 6



SYSTEMS OF EQUATIONS

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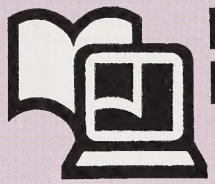
Welcome



Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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Mathematics 10 Student Module Unit 6 Systems of Equations Alberta Distance Learning Centre ISBN No. 0-7741-0751-0

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General Information

This information explains the basic layout of each booklet.

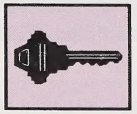
- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

Visual Cues

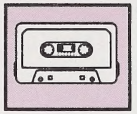
Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.

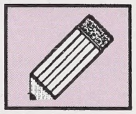
What You Already Know
- reviewing what you already know



Key Idea
- flagging important ideas

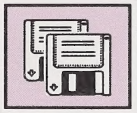


Audiotape
- learning by listening to an audiotape



Review
- studying previous concepts

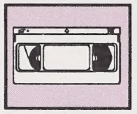
Another View
- exploring different perspectives



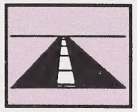
Computer Software
- learning by using computer software

Introduction
- introducing the unit

Solutions
- correcting the activities



Videotape
- learning by viewing a videotape



What Lies Ahead
- previewing the unit

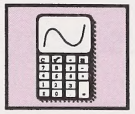
Extra Help
- providing additional study

Print Pathway
- choosing a print alternative

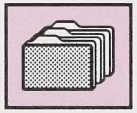
Exploring the Topic
- actively learning new concepts

Extensions
- going on with the topic

Calculator
- using your calculator



Graphing Calculator
- using your graphing calculator



What You Have Learned
- summarizing what you have learned

Mathematics 10

Course Overview

Mathematics 10 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Operations on Polynomials	14%
Unit 3 Equations and Inequalities	10%
Unit 4 Factoring Polynomials	13%
Unit 5 Coordinate Geometry	19%
Unit 6 Systems of Equations	**10%**
Unit 7 Trigonometry	11%
Unit 8 Statistics	13%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Systems of Equations

This unit covers topics dealing with systems of equations. Each topic contains explanations, examples, and activities to assist you in understanding systems of equations. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 6 Systems of Equations

Contents at a Glance

Value		
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	What You Already Know	5
	Review	8
40%	**Topic 1:** Solving Systems of Equations Graphically	9
	• Introduction • Extra Help • What Lies Ahead • Extensions • Exploring Topic 1	
60%	**Topic 2:** Solving Systems of Equations Algebraically	38
	• Introduction • Extra Help • What Lies Ahead • Extensions • Exploring Topic 2	
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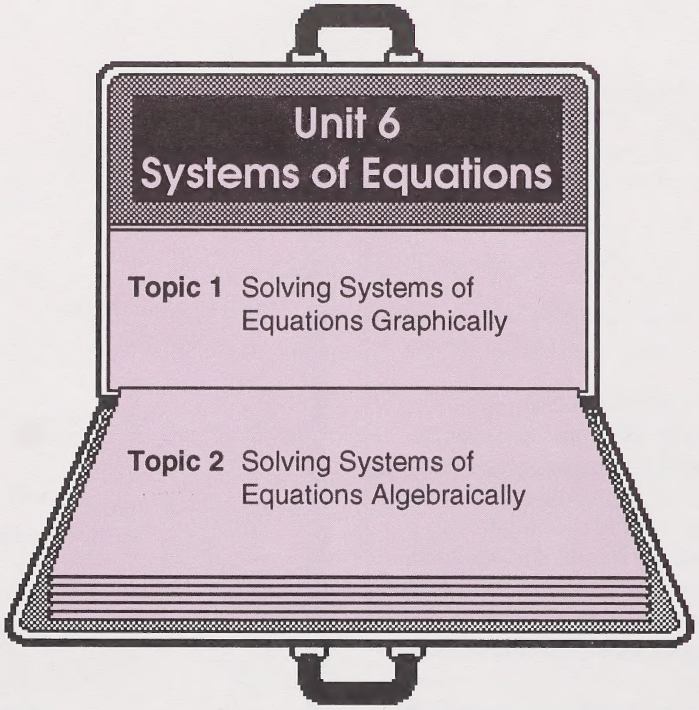
	• Appendix A • Appendix B	

Systems of Equations

You have learned to solve equations in one variable. If there are two variables, then you need two equations to form a system of equations; then, solve the system of equations in the two variables. If there are three variables, you need three equations. If there are four variables, you need four equations, and so on. A system of equations is a very useful tool to solve more complicated problems that will arise later in your study of mathematics. However, in this unit, you are going to investigate sytems of linear equations in two variables only.


Unit 6
Systems of Equations
Topic 1 Solving Systems of Equations Graphically
Topic 2 Solving Systems of Equations Algebraically

What You Already Know

Refresh your memory!

Review the solutions to the following equations.

- $x+3=4$

$x+3-3=4-3$ (Subtract 3 from both sides.)

$x=1$

Check your answer.

LS	RS
$x+3$	4
$(1)+3$	4
4	4
LS =	RS

- $3x=9$

$\frac{3x}{3}=\frac{9}{3}$ (Divide both sides by 3.)

$x=3$

Check your answer.

LS	RS
$3x$	9
$3(3)$	9
9	9
LS =	RS

- $3x+2=11$

$3x+2-2=11-2$ (Subtract 2 from both sides.)

$3x=9$

$\frac{3x}{3}=\frac{9}{3}$ (Divide both sides by 3.)

$x=3$

Check your answer.

LS	RS
$3x+2$	11
$3(3)+2$	11
$9+2$	11
11	11
LS =	RS

- $\frac{x}{2}=\frac{12}{3}$

$3(x)=2(12)$ (Cross multiply.)

$3x=24$

$\frac{3x}{3}=\frac{24}{3}$ (Divide both sides by 3.)

$x=8$

Check your answer.

LS	RS
$\frac{x}{2}$	$\frac{12}{3}$
$\frac{(8)}{2}$	4
4	4
LS =	RS

- $4x+2=3x$

$4x+2-2=3x-2$ (Subtract 2 from both sides.)

$4x=3x-2$

$4x-3x=3x-2-3x$ (Subtract $3x$ from both sides.)

$x=-2$

Check your answer.

LS	RS
$4x+2$	$3x$
$4(-2)+2$	$3(-2)$
$-8+2$	-6
-6	-6
LS =	RS

- $2(3x+3)=-3$

$2(3x)+2(3)=-3$ (Use the distributive property.)

$6x+6=-3$

$6x+6-6=-3-6$

$6x=-9$

$x=\frac{-9}{6}$

$=\frac{-3}{2}$

Check your answer.

LS	RS
$2(3x+3)$	-3
$2\left[3\left(\frac{-3}{2}\right)+3\right]$	-3
$2\left(\frac{-9}{2}+3\right)$	-3
$2\left(\frac{-9}{2}+\frac{6}{2}\right)$	-3
$2\left(\frac{-3}{2}\right)$	-3
-3	-3

LS = RS

- $4x+6=24x-4$

$4x-24x=-4-6$ (Transpose.)

$-20x=-10$ (Collect the similar terms.)

$x=\frac{1}{2}$

Check your answer.

LS	RS
$4x+6$	$24x-4$
$4\left(\frac{1}{2}\right)+6$	$24\left(\frac{1}{2}\right)-4$
$2+6$	$12-4$
8	8

LS = RS

Now that you have looked at material you studied previously, go to the **Review** to confirm your understanding of this material.

Transpose means to move terms from one side of the equal sign to the other. Remember to change the sign of the terms you move.

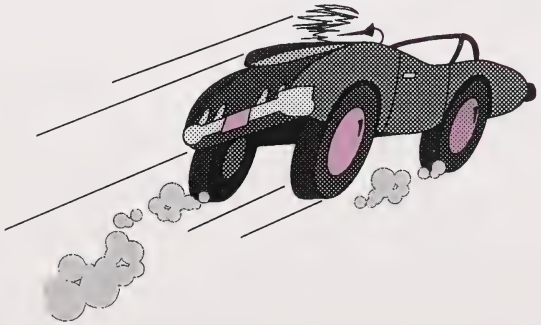
Review

Try the following review questions.

Solve and check each of the following equations.

1. $3x-4=23$
2. $2(3x-5)=32$
3. $4(x-3)+9x=-38$
4. $7m+2=5m+18$
5. $3(2r+4)=4(5r-4)$
6. $x-5=8-2(x+2)$
7. $2(5x-11)-6=3(x-7)-15$

Now go to the **Review** solutions in **Appendix A**.



Topic 1 Solving Systems of Equations Graphically

Introduction

Many scientific or business situations can be illustrated by linear equations with two variables. A linear equation with two variables can be illustrated by a straight line on a coordinate plane. For example, a publisher can use a straight line on a coordinate plane to illustrate the relation between income and the number of books sold. You can also graph another straight line on the same axes that illustrates the relation between cost and number of books sold. The point of intersection of the two straight lines represents the break-even point, and the two linear equations form a system of equations. How do you graph those straight lines? How do you find the point of intersection? These are the things that you are going to learn in this topic.

What Lies Ahead

Throughout this topic you will learn to

1. solve and classify a system of two linear equations by graphing
2. classify a system of two linear equations as dependent, independent, or inconsistent according to the slope and the y-intercepts of the equations
3. use a graphing calculator to solve a system of equations

Now that you know what to expect, turn the page to begin your study of solving systems of equations graphically.

Exploring Topic 1

Activity 1

Solve and classify a system of two linear equations by graphing.

A salesperson at a wholesale videocassette store can work for salary plus commission or commission only. The monthly pay is represented by the following equations.

The equation for salary plus commission is as follows:

$$y = 1500 + 0.25x$$

The equation for commission only is as follows:

$$y = 0.5x$$

In both equations, y is the monthly pay and x is the number of tapes sold.

How does a salesperson decide whether to choose commission only or salary plus commission?

$y = 1500 + 0.25x$ means \$1500 fixed salary plus \$0.25 per tape sold.

$y = 0.5x$ means no fixed salary. \$0.50 is received for each tape sold.

The salesperson may illustrate the two equations graphically. This is shown as follows:

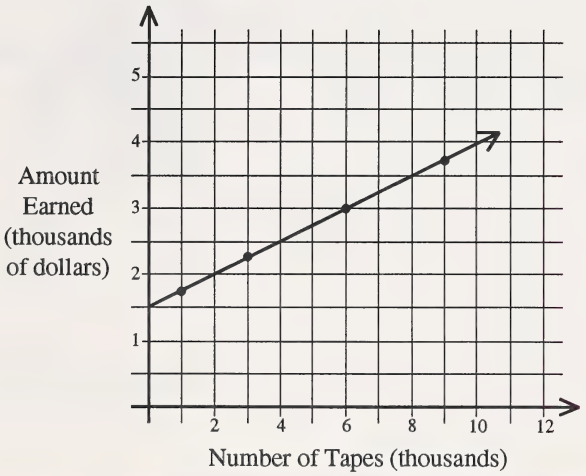
The graph for salary plus commission is as follows:

$$y = 1500 + 0.25x$$

Make a table of values.

x	y
1000	1750
3000	2250
6000	3000
9000	3750

Construct the graph.



Theoretically, x can be any number. Since this is a real life situation, the number of tapes has to be a whole number. Choose numbers that are easy to calculate and substitute them into the equation to find y or vice versa.

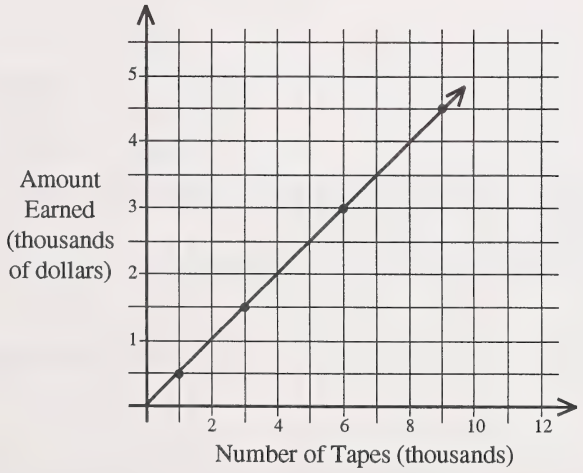
The graph for commission only is as follows:

$y = 0.5x$

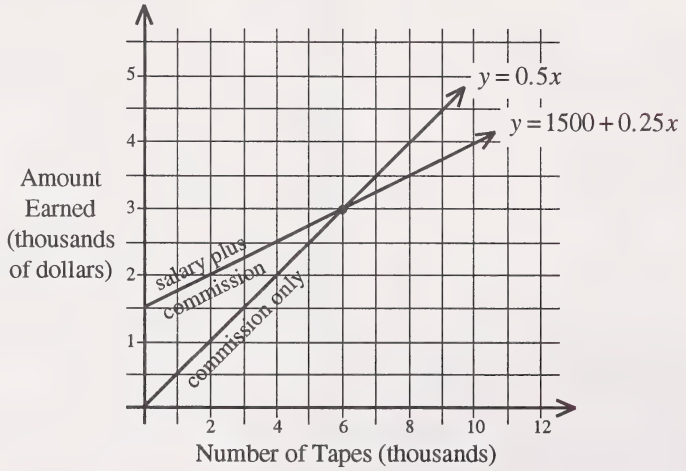
Make a table of values.

x	y
1000	500
3000	1500
6000	3000
9000	4500

Construct the graph.

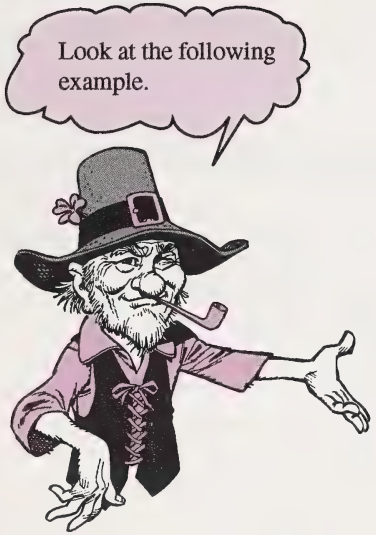


When the two lines are graphed on the same axes, they intersect each other. Look at the following graph.



In this case, the point of intersection is (6000, 3000). This indicates that the salesperson would get $3000 if the total sales are 6000 tapes regardless whether working for commission only or salary plus commission. If sales are less than 6000 tapes, the person who works for salary plus commission would get more money. If sales are greater than 6000 tapes, then the person who works for commission only would get more money. This graph helps you to make comparisons and answer a lot of questions.

These two equations put together form a system of equations. Is there always a point of intersection? Is it possible to have more than one point of intersection?



Example 1

Solve the following linear system graphically.

$2x+y=5$ (1)
$x-y=4$ (2)

Solution:

Make a table of values for each question.

$2x+y=5$

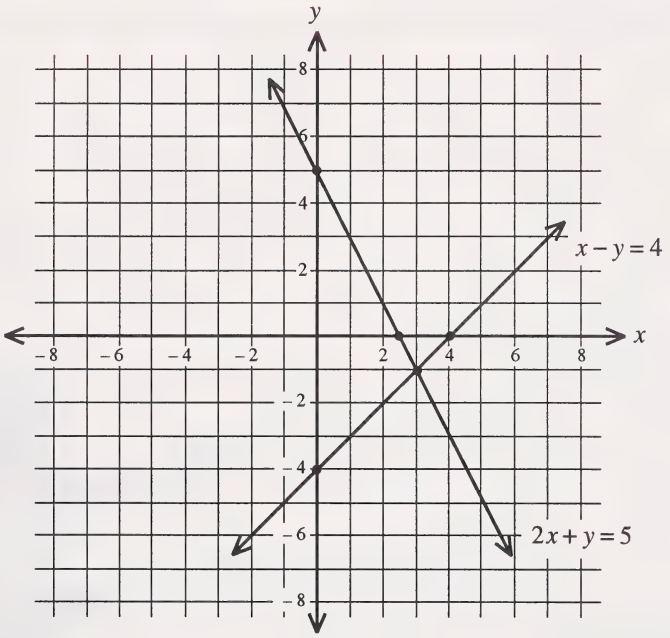
x	y
0	5
2	1
3	−1

$x-y=4$

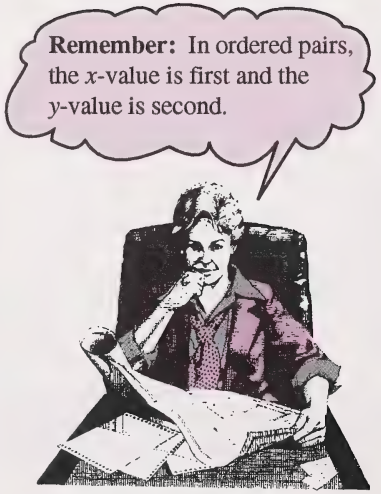
x	y
4	0
0	−4
3	−1

The equations in a system are numbered for easier reference.

A system of equations is also called simultaneous equations.



The graph shows that the point of intersection is $(3, -1)$. There is only one point of intersection.



Check the solution. Substitute $x = 3$ and $y = -1$ into both equations.

Check (1).

LS	RS
$2x + y$	5
$2(3) + (-1)$	5
$6 - 1$	5
5	5
LS =	RS

Check (2).

LS	RS
$x - y$	4
$3 - (-1)$	4
4	4
LS =	RS

The solution $(3, -1)$ is correct. $(3, -1)$ is the only pair of numbers that satisfies both equations.

A system of equations that has one solution is called an **independent (or intersecting) system of equations.**

Example 2

Solve the following linear system graphically.

$2x - y = 6$ (1)
$6x - 3y = 18$ (2)

Solution:

$2x - y = 6$

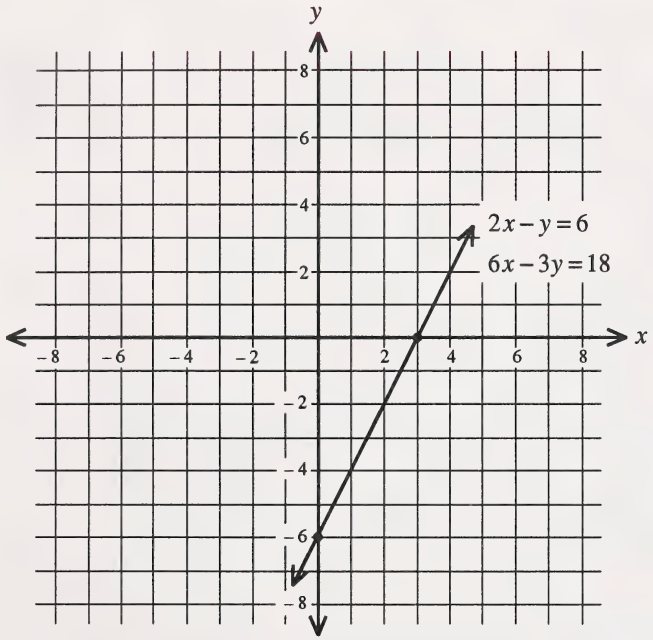
x	y
0	-6
3	0
4	2

$6x - 3y = 18$

x	y
3	0
0	-6
4	2

LS = Left Side
RS = Right Side

Construct the graph.



It looks like the two lines coincide. It means that the intersection of these two lines is the whole line. Therefore, there are an infinite number of points of intersection; thus, there are an infinite number of solutions.

A system of equations that has an infinite number of solutions is called a **dependent (or coincident) system of equations**.

If one equation is a multiple of the other, these two equations form a dependent system of equations.

Note that if each term of (1) is multiplied by three, you get (2).

$$2x - y = 6 \quad (1)$$
$$3(2x) - 3(y) = 3(6) \quad \text{(Multiply each term by 3.)}$$
$$6x - 3y = 18 \quad \text{(This is identical to (2).)}$$

Would it be possible that a system of equations has no solution? Look at Example 3.

Example 3

Solve the following linear system graphically.

$2x - 3y = 9$ (1)
$4x - 6y = 6$ (2)

Solution:

Make a table of values for each equation.

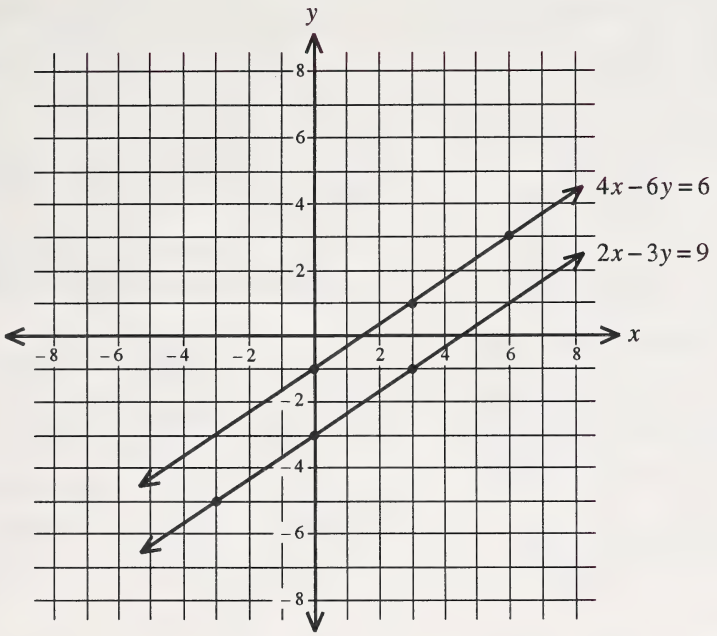
$2x - 3y = 9$

x	y
0	−3
3	−1
−3	−5

$4x - 6y = 6$

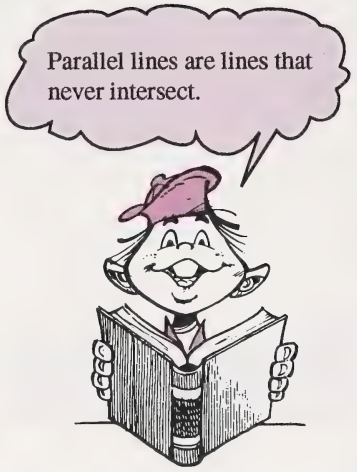
x	y
0	−1
3	1
6	3

Construct the graph.



The graphs of the equations are two parallel lines because they do not intersect. Therefore, these two lines do not have a point of intersection, and the system does not have a solution.

A system of equations that has no solution is called an **inconsistent (or parallel) system of equations**.



The next example is a word problem based on a system of equations. The coefficients are not integers.

Example 4

There are two numbers. If the first number is added to $\frac{1}{2}$ of the second number, the sum is 4. If $\frac{2}{3}$ of the first number is added to $\frac{1}{4}$ of the second number, the sum is $\frac{5}{2}$. Find the two numbers.

Solution:

Let x be the first number and y be the second number.

$$x+\frac{1}{2}y=4 \quad (1)$$

$$\frac{2}{3}x+\frac{1}{4}y=\frac{5}{2} \quad (2)$$

Multiply every term of the first equation by two to clear the denominator.

$$2(x)+2\left(\frac{1}{2}y\right)=2(4)$$

$$2x+y=8 \quad (3)$$

Since $2x+y=8$ is a multiple of $x+\frac{1}{2}y+4$, the **two equations represent the same straight line**; thus, they are **equivalent**.

Make a table of values.

$$2x+y=8$$

x	y
0	8
1	6
2	4

Multiply every term of the second equation by 12.

$$(12)\left(\frac{2}{3}x\right)+(12)\left(\frac{1}{4}y\right)=(12)\left(\frac{5}{2}\right)$$

$$8x+3y=30 \quad (4)$$

$8x+3y=30$ is equivalent to (2) because it is a multiple of it.

Make a table of values.

$$8x+3y=30$$

x	y
0	10
3	2
1	$7\frac{1}{3}$

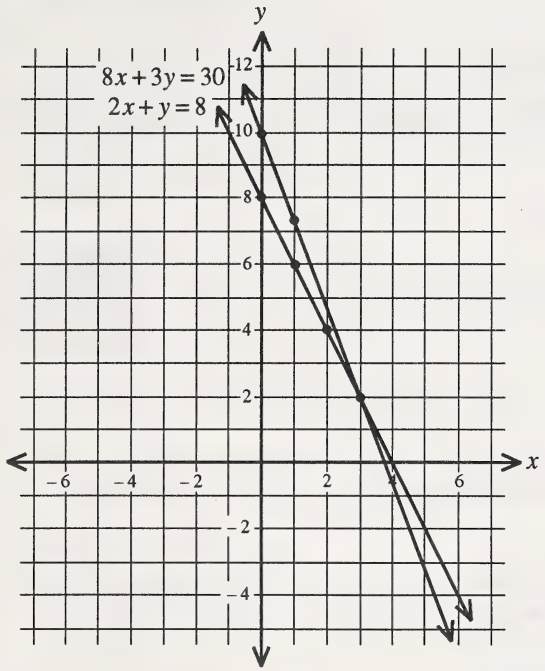
12 is the lowest common denominator of 2, 3, and 4.

$2x+y=8$ (3)

$8x+3y=30$ (4)

Since this is equivalent to the original system of equations, the intersection of these two lines is the same as the intersection of the two original lines. The intersection will represent the two numbers that you are looking for.

Construct the graph.



The point of intersection is $(3, 2)$. Thus, the two numbers are $x=3$ and $y=2$.

Check $x=3$ and $y=2$ in (1).

LS	RS
$x+\frac{1}{2}y$	4
$3+\frac{1}{2}(2)$	4
$3+1$	4
4	4

LS = RS

Check $x=3$ and $y=2$ in (2).

LS	RS
$\frac{2}{3}x+\frac{1}{4}y$	$\frac{5}{2}$
$\frac{2}{3}(3)+\frac{1}{4}(2)$	$\frac{5}{2}$
$2+\frac{1}{2}$	$\frac{5}{2}$
$2\frac{1}{2}$	$\frac{5}{2}$
$\frac{5}{2}$	$\frac{5}{2}$

LS = RS

In an ordered pair, the first number is x and the second number is y.

Use the original equations when checking.

If you have access to a graphing calculator or a computer graphing program, use it to do the graphing sections indicated by the graphing calculator icon in this unit. You may also use the graphing utility to check the graphs that have been used in the print part of this unit. Some of the graphs require a different range setting in order for the graph to appear on the screen.

Go to **Mathematics 10, Unit 5** for more information on how to graph equations on a graphing calculator, and how to set the range.

Now try some examples using the graphing utility.

Exploring Mathematics with a Graphing Calculator

Example 5

Use a graphing calculator or a computer graphing program to graph the following systems of equations. Classify as either independent (intersecting), dependent (coincident), or inconsistent (parallel).

- $3x + y = 4$ (1)
  $x + y = 2$ (2)

Solution:

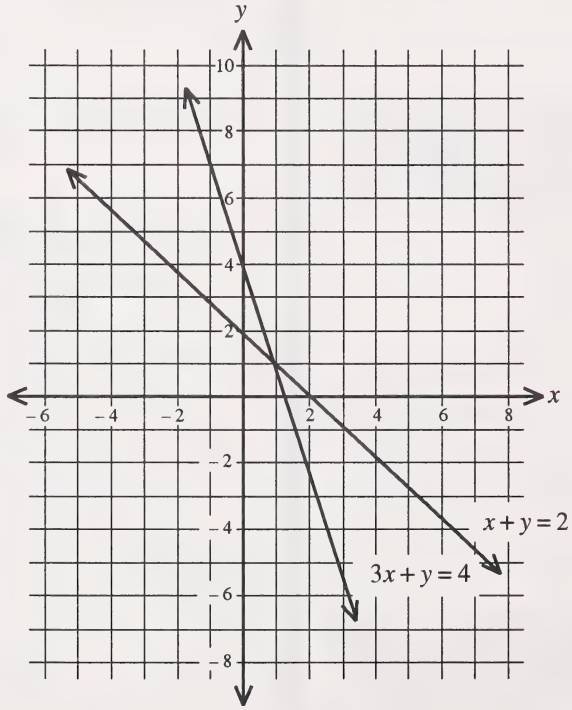
Change (1) to $y = mx + b$ form. (This may not be necessary for the computer graphing program.)

$y = -3x + 4$

Change (2) to $y = mx + b$ form.

$y = -x + 2$

Using a range of -10 to $+10$ on each axis, graph the equations. Your graph should look like the following.



These equations are independent or intersecting.

- $2x-4y=6$ (1)
  $x-y=3$ (2)

Solution:

Change (1) to $y=mx+b$ form.

$$-4y=-2x+6$$

$$y=\frac{-2x}{-4}+\frac{6}{-4}$$

$$y=\left(\frac{1}{2}\right)x-\left(\frac{3}{2}\right) \text{ or } y=0.5x-1.5$$

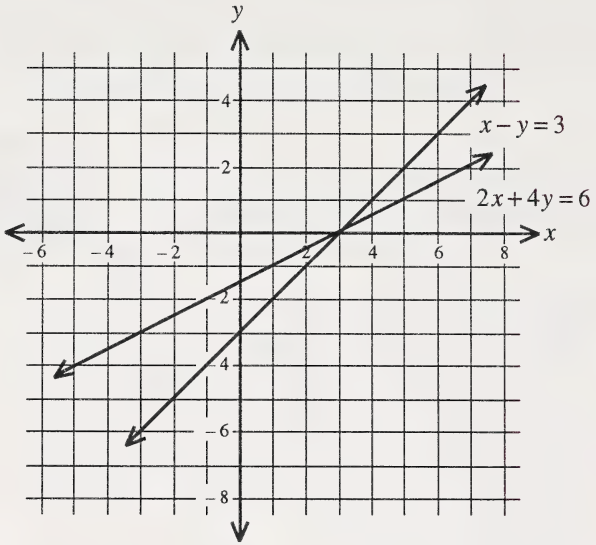
Note: Use brackets to graph equations that have fractions. For example, $y=\left(\frac{1}{2}\right)x-\left(\frac{3}{2}\right)$.

Change (2) to $y=mx+b$ form.

$$-y=-x+3$$

$$y=x-3$$

Graph the equations. Your graph should be similar to the following:



These equations are independent or intersecting.

• $2x-y=5$ (1)

$2x-y=-3$ (2)

Solution:

Change (1) to $y=mx+b$ form.

$-y=-2x+5$

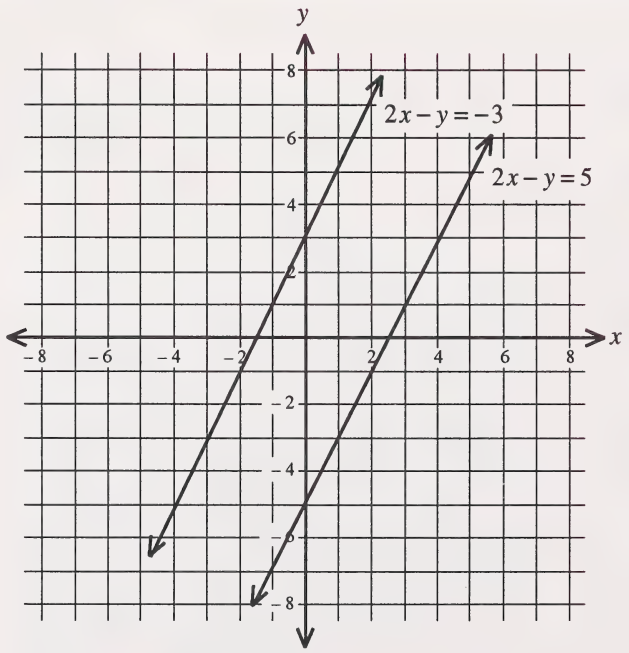
$y=2x-5$

Change (2) to $y=mx+b$ form.

$-y=-2x-3$

$y=2x+3$

Graph the equation.



The equations are inconsistent or parallel.

For questions 1 through 4, do either a or b. Graph paper is provided in **Appendix B**.

1. Use the method of graphing to solve each of the following systems of equations. Check your solution.

   a. $5x-2=4$ (1)
      $x-y=2$ (2)

   b. $8x-y=1$ (1)
      $x+2y=15$ (2)

2. Use the method of graphing to solve each of the following system of equations.

   a. $3x-y=7$ (1)
      $6x-2y=8$ (2)

   b. $x-3y=9$ (1)
      $3x-9y=18$ (2)

3. Use the method of graphing to solve the following systems of equations.

   a. $4x-10y=8$ (1)
      $6x-15y=12$ (2)

   b. $2x-4y=6$ (1)
      $3x-6y=9$ (2)

4. Use the method of graphing to solve the following systems of equations.

   a. $x-\frac{3}{2}y=7$ (1)
      $\frac{1}{2}x-1y=\frac{7}{2}$ (2)

   b. $x+\frac{1}{3}y=3$ (1)
      $\frac{2}{5}x+y=\frac{19}{5}$ (2)

5. For the equations you solved in questions 1 to 3, classify as independent, inconsistent, or dependent of equations.

6. One third of the sum of two numbers is 1. If three times the first number is added to one-half of the second number, the result is 4. Graph the resulting equations to find the numbers.

If you have access to a graphing calculator or a computer graphing program, do question 7.

7. Use a graphing calculator or a computer graphing program to graph the following systems of equations. Classify them as either independent (intersecting), dependent (coincident), or inconsistent (parallel).

   a. $2x-y=7$ (1)
      $x+y=4$ (2)

   b. $3x+2y=11$ (1)
      $3x+2y=7$ (2)

   c. $5x-y=3$ (1)
      $10x-2y=6$ (2)

   d. $3x-5y=-10$ (1)
      $3x+2y=5$ (2)

For solutions to **Activity 1**, turn to the **Appendix A**, **Topic 1**.

Activity 2

Classify a system of two linear equations as dependent, independent, or inconsistent according to the slopes and the y-intercepts of the equations.

Another way to determine whether a system is independent, dependent, or inconsistent is by writing each equation of the system in slope y-intercept form.

Example 6

Determine whether the following system is independent, dependent, or inconsistent.

$x+3y=1$ (1)
$3x-y=5$ (2)

Solution:

$x+3y=1$ (Subtract x from both sides.)

$3y=-x+1$

$y=-\frac{1}{3}x+\frac{1}{3}$ (Divide every term by 3 to make the equation into $y=mx+b$ form.)

The slope m (the coefficient of x) is $-\frac{1}{3}$, and the y-intercept b is $\frac{1}{3}$.

Change the equations to $y=mx+b$ form. Recall that this form is known as the slope y-intercept form.

$3x - y = 5$ (Subtract $3x$ from both sides.)

$-y = -3x + 5$

$y = 3x - 5$ (Divide every term by -1.)

The slope m is 3, and the y-intercept b is -5.

The two slopes are different. Therefore, this system is independent or intersecting.

If the slopes are different, the system is **independent**.

Example 7

Determine whether the following system of equations is independent, dependent, or inconsistent.

$2x - 3y = 5$ (1)

$4x - 6y = 7$ (2)

Solution:

Change both equations to $y = mx + b$ form.

$2x - 3y = 5$ (Subtract $2x$ from both sides.)

$-3y = -2x + 5$

$y = \frac{2}{3}x - \frac{5}{3}$ (Divide both sides by -3.)

The slope m is $\frac{2}{3}$, and the y-intercept b is $-\frac{5}{3}$.

$4x - 6y = 7$ (Subtract $4x$ from both sides.)

$-6y = -4x + 7$

$y = \frac{4}{6}x - \frac{7}{6}$ (Divide both sides by -6.)

$y = \frac{2}{3}x - \frac{7}{6}$

The slope m is $\frac{2}{3}$, and the y-intercept b is $-\frac{7}{6}$.

The slopes are equal and the y-intercepts are different. Therefore, this system is inconsistent or parallel.

If the slopes are equal and the y-intercepts are different, the system is **inconsistent**.

If the slopes are equal, then the two lines are parallel. If the y-intercepts are different, then they are two separate lines that do not intersect each other. Therefore, there is no solution (point of intersection) for this system of equations.

Example 8

Determine whether the following sytem of equations is independent, dependent, or inconsistent.

$2x+6y=4$ (1)
$3x+9y=6$ (2)

Solution:

$2x+6y=4$ (Subtract $2x$ from both sides.)
$6y=-2x+4$
$y=-\frac{2}{6}x+\frac{4}{6}$ (Divide both sides by 6.)
$y=-\frac{1}{3}x+\frac{2}{3}$

The slope m is $-\frac{1}{3}$, and the y-intercept b is $-\frac{2}{3}$.

$3x+9y=6$ (Subtract $3x$ from both sides.)
$9y=-3x+6$
$y=-\frac{3}{9}x+\frac{6}{9}$ (Divide both sides by 9.)
$y=-\frac{1}{3}x+\frac{2}{3}$

The slope m is $-\frac{1}{3}$, and the y-intercept b is $\frac{2}{3}$.

The slopes and the y-intercepts are equal. Therefore, this system is dependent or coincident.

Equation (1) $\times \frac{3}{2}$ = Equation (2)
or
Equation (2) $\times \frac{2}{3}$ = Equation (1)

If the slopes and the y-intercepts are both the same, then the two lines coincide. There is actually only one line. Since this line represents the intersection of two equal lines, there are an infinite number of solutions for this system.

If the slopes and the y-intercepts are both the same, the system is **dependent**.

For questions 1 to 3, do either a or b.

1. Use the method of writing each equation of the system in slope y-intercept form to classify each of the following systems.

   a. $5x-20y=10$ (1)
      $3x-12y=6$ (2)

   b. $2x+4y=6$ (1)
      $7x+14y=21$ (2)

2. Use the method of writing each equation of the system in slope y-intercept form to classify each of the following systems.

   a. $7x-5y=13$ (1)
      $14x-10y=7$ (2)

   b. $5x+8y=11$ (1)
      $10x+16y=14$ (2)

3. Use the method of writing each equation of the system in slope y-intercept form to classify each of the following systems.

   a. $3x-8y=12$ (1)
      $2x-8y=4$ (2)

   b. $6x-7y=15$ (1)
      $3x-4y=12$ (2)

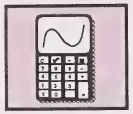
For solutions to **Activity 2**, turn to **Appendix A**, **Topic 1**.

Remember the following:

- For independent systems, the slopes are different.
- For inconsistent systems, the slopes are equal and the y-intercepts are different.
- For dependent systems, the slopes and the y-intercepts are equal.

Activity 3

Use a graphing calculator to solve a system of equations.



If you have access to a graphing calculator or a computer graphing program, do this activity.

In this activity you will learn to use a graphing calculator to solve systems of equations. If you are using a computer graphing program, consult the information that came with the program on how to use it to solve a system of two linear equations.

In **Unit 5** you learned how to enter equations into the graphing calculator. To enter two equations and have them graphed on the same set of axes, simply enter the equations one after the other. For the Casio *fx*-7700G graphing calculator, each equation is graphed separately. For the TI-81 graphing calculator, the two equations can be entered first; then, the equations are graphed at once or each equation can be entered and graphed separately.

One key you must pay special attention to for solving systems of equations is TRACE. The TRACE function is used to find the coordinates of the point of intersection of the two equations. On the Casio *fx*-7700G, the TRACE function is initiated by pressing SHIFT and F1. Not all Casio graphing calculators are the same. Consult your manual regarding your particular calculator. On the TI-81, simply press TRACE. In either case this places a flashing curser on the screen along with the coordinates for x and y. To move the curser along the graph, press ▶ or ◀.

Study the following examples which demonstrate how to use a graphing calculator to solve a system of two equations in two unknowns.

Example 9

Find the solution to the following system of equation using a graphing calculator.

$x+y=7$ (1)
$3x-2y=1$ (2)

Solution:

Change (1) to $y=mx+b$ form.

$$y=-x+7$$

Change (2) to $y=mx+b$ form.

$$-2y=-3x+1$$
$$y=\frac{3x}{2}-\frac{1}{2}$$
$$y=1.5x-0.5$$

If you have the Casio *fx*-7700G, follow these steps.

Step 1: Turn the calculator on. Check the range. If it is not at -10 to $+10$ for both x and y with a scale of 1, then set the range to these values. Refer back to **Unit 5** for setting the range, or consult the calculator manual.

Step 2: Before you enter the equations, press G↔T. This will display the axes. If there are any previous graphs present, clear them by pressing SHIFT, F5, and EXE. This will clear the graphs and take you back to the text screen.

Step 3: Clear this screen with the AC (all clear) key. Now press GRAPH. Graph Y= should appear on the screen.

Step 4: Enter the first equation and press EXE. Press GRAPH again after the first graph has been completed. Enter the second equation and press EXE. Remember, you only have to enter the $mx+b$ part of the equation. Now you should have two lines graphed as shown following the steps to the TI-81 calculator.

Step 5: Now press SHIFT and F1 to get the TRACE function. The flashing cursor should appear on the second equation, and the values of the x- and y-coordinates should appear at the bottom. If you have let the calculator idle until it has shut itself off, you will get the graphs back when you turn it back on, but you will not be able to use the trace function. To use the trace function, you will have to re-enter the graphs.

Step 6: To move the cursor to the point of intersection, use ▶ or ◀. Once you have the cursor as close to the point of intersection as possible, read the values of the x- and y-coordinates. Round the values to the nearest integer. In this equation, the x-value is 3 and the y-value is 4.

Note: The ZOOM function, which allows you to find the x- and y-intercepts more accurately, cannot be used with two functions on the Casio *fx*-7700G.

Step 7: Clear the graphing screen, as given in Step 2, and clear the text screen by pressing AC. Now you are ready to graph a new pair of equations.

If you have the TI-81, follow these steps.

Step 1: Turn the calculator on. Check the range. If it is not at -10 to $+10$ for both x and y with the scale of 1, then set the range to these values. Refer back to **Unit 5** for setting the range or consult the calculator manual.

Step 2: Press Y=. This will take you to the screen where you will enter the equations. Enter the first equation and press ENTER. Be sure to use the negative (–) key for $-x$ not the subtraction key. Now enter the second equation and press ENTER.

Step 4: Use the TRACE function to find the point of intersection. Press TRACE and move the flashing cursor to the point of intersection using ▶ or ◀. Once you have the cursor on the point of intersection, you can read the values of x and y and round to the nearest integer; or you can use the ZOOM function to get a more accurate positioning of the cursor.

Step 5: To use the ZOOM function, press ZOOM when the cursor is at the point of intersection. You will get a screen with eight choices on it. Use ▲ or ▼ to move to the desired choice. To find a more accurate point of intersection, use the BOX function, which is choice one.

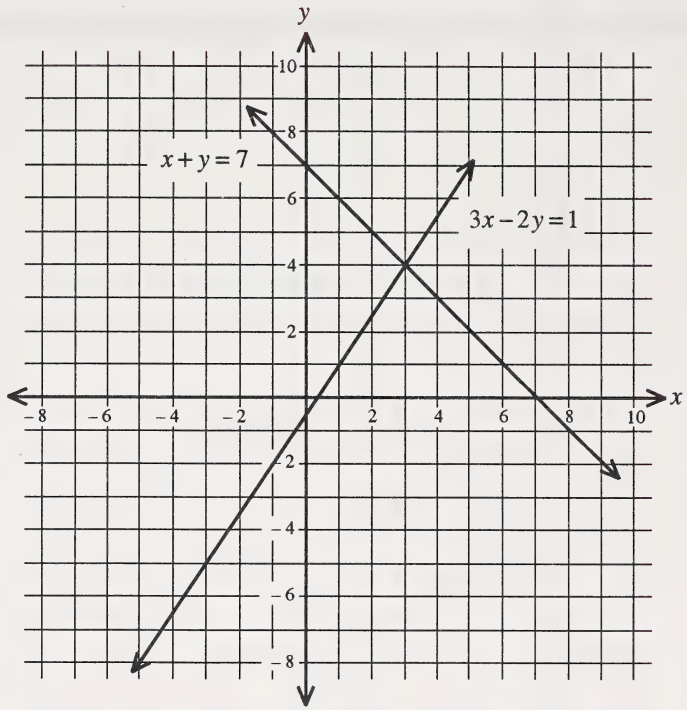
Step 6: To use the BOX function press ENTER and move the cursor a little right or left and up or down from the point of intersection. Press ENTER. This will mark a spot away from the point of intersection. Now move the cursor again. This time you should notice a line drawn as you move the cursor. Stop when you are on the other side of the point of intersection. Now move the flashing cursor past the point of intersection again. You should notice that a box has been built around the point of intersection. Make the box small (about 1 cm by 1 cm), but be sure the point of intersection is in it.

Step 7: Press ENTER and an enlarged graph of the point of intersection should appear on the screen.

Step 8: Press TRACE and move the cursor to the point of intersection. You may use the BOX function several times. Simply press ZOOM and ENTER, and build the little box around the point of intersection again. If you keep repeating the use of the ZOOM function, you will get a more accurate value of x and y each time. After six to ten repetitions of the ZOOM function, the calculator will give integer values for x and y.

Step 9: Before you try another set of equations after using the ZOOM function, be sure to go back to the original graph. Do this by pressing ZOOM again and moving down to Standard, which is number six, and press ENTER. This will redo the original graph and reset the range to the original values. Otherwise you will have to reset the range each time you want to graph another set of equations.

Step 10: Press Y= and clear each of the old equations with CLEAR. Now you are ready to graph another set of equations.



The solution to this system of equations is $x = 3$ and $y = 4$ or $(3, 4)$.

Try one more example.

Example 10

Find the solution to the following system of equations using a graphing calculator.

$7x-4y=26$ (1)
$3x+4y=-6$ (2)

Solution:

Change (1) to $y=mx+b$ form.

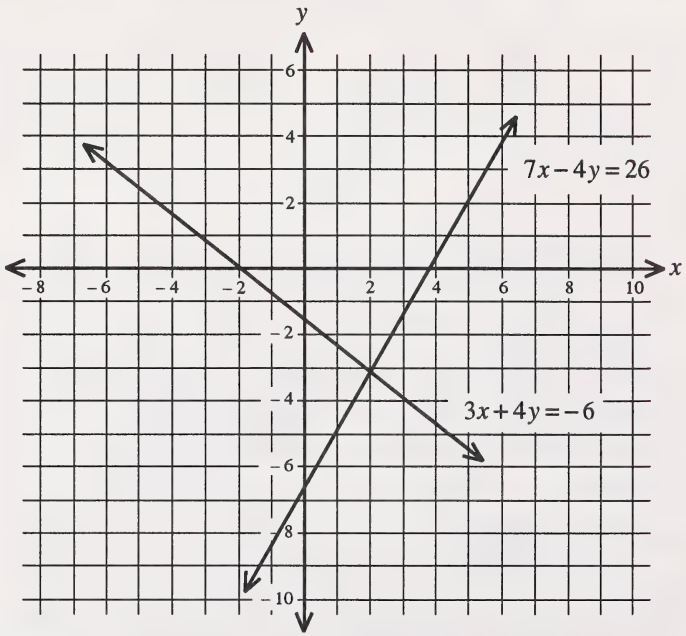
$$-4y=-7x+26$$
$$y=\frac{7x}{4}-\frac{26}{4}$$

Change (2) to $y=mx+b$ form.

$$4y=-3x-6$$
$$y=-\frac{3x}{4}-\frac{6}{4}$$

Enter the equations into your calculator. Refer to Example 9 if you are having difficulty. If you are using the TI-81 graphing calculator, you should find that it will give you integer coordinates by using the TRACE function the first time. Thus, there is no need to use the ZOOM function here.

The graph should look as follows:



The point of intersection is $x=2$ and $y=-3$. Therefore, the solution to the systems of equations is $(2,-3)$.

Find the solution to the following systems of equations using a graphing calculator or a computer graphing program.

1. $4x-3y=20$ (1)
   $6x-3y=24$ (2)

2. $-5x+2y=-1$ (1)
   $5x-4y=-13$ (2)

3. $2x-5y=29$ (1)
   $7x-3y=0$ (2)

4. $4x+y=-11$ (1)
   $3x-5y=9$ (2)

5. $x-3y=4$ (1)
   $2x-6y=8$ (2)

For solutions to **Activity 3**, turn to **Appendix A**, **Topic 1**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

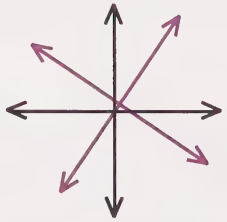
Extra Help

The following diagram shows the three kinds of systems of equations.

Systems
- Consistent (has solution(s))
  - Independent (one solution)
  - Dependent (many solutions)
- Inconsistent (no solution)

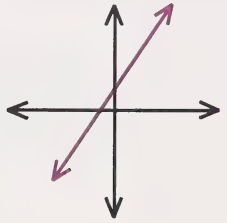
The characteristics of an **independent system** are as follows:

- The graph has two intersecting lines.
- There is one point of intersection.
- The point of intersection is the solution.
- The slopes of the two lines are different.



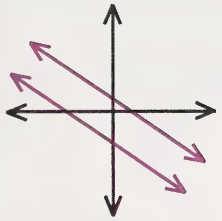
The characteristics of a **dependent system** are as follows:

- The graph has one straight line (two lines coincide).
- There are an infinite number of points of intersection.
- The whole line is the solution.
- The slopes and the y-intercepts of the two lines are equal.



The characteristics of an **inconsistent system** are as follows:

- The graph has two parallel lines.
- There is no point of intersection.
- There is no solution.
- The slopes of the two lines are equal but the y-intercepts are different.



If you had trouble writing an equation in slope y-intercept form, do question 1 and any two of question 2.

1. Rewrite the equation $3x-5y=7$ in slope y-intercept form.

   Solution:

   To write an equation in slope y-intercept form, you have to isolate y on the left side.

   $3x-5y=7$

   Step 1: Remove $3x$ from the left side by _____

   _____.

   $3x-5y-3x=7-3x$

   $-5y=-3x+7$

   Step 2: Isolate y by _____

   _____.

   $$\frac{-5y}{(\ )}=\frac{-3}{(\ )}x+\frac{7}{(\ )}$$

   Step 3: Simplify the equation.

   $$y=\frac{3}{5}x-\frac{7}{5}$$

   Why is $\frac{3}{5}$ positive?

   The slope is $\frac{3}{5}$ and the y-intercept is $-\frac{7}{5}$.

Remember that the coefficient of y must be positive 1. Only one y should be remaining on the left side.

2. Write the following in slope y-intercept form. State the slope and the y-intercept.

   a. $3x-11y=9$

   b. $5x+7y=19$

   c. $8x-9y=0$

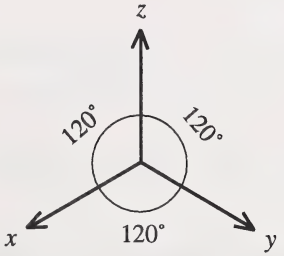
   d. $2x-12y=-3$

For solutions to **Extra Help**, turn to **Appendix A**, **Topic 1**.

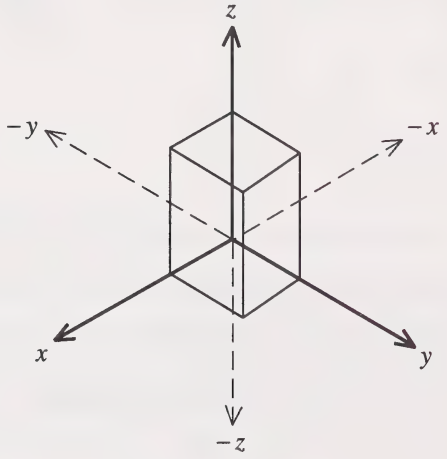
Extensions

Equations in this topic are linear equations with two variables. They represent straight lines in two dimensional space. What kind of graph would it represent if the equation has three variables? How do you draw the graph?

If an equation has three variables. It represents a plane in three-dimensional space. Assume that x, y, and z are the three variables. You need the x-, y-, and z-axes. In order to draw the three positive axes on two dimensional paper, you can make the axes appear at 120° from each other as shown in the following diagram. Look at the corner of a room where the floor meets the walls. Looking at the corner makes the angles appear to be 120°.



This graph is one perspective and the **real** axes are 90° from each other. The intersection of the x-, y-, and z-axes is at the point (0, 0, 0). Negative values are plotted on dotted line extensions of the three axes as shown.



When plotting a point (x, y, z) in three-space, move a distance parallel to the appropriate axes. When plotting a plane in three-space, find the x-, y-, and z-intercepts and join the three intercepts. The triangle formed represents the plane determined by these three points.

Example 11 shows you how to do this.

Example 11

$2x+y-z=4$

Solution:

Let $x=0$ and $y=0$.

Then, $0+0-z=4$

$z=-4$

The z-intercept is $(0,0,-4)$.

Let $y=0$ and $z=0$.

Then, $2x+0-0=4$

$2x=4$

$x=2$

The x-intercept is $(2,0,0)$.

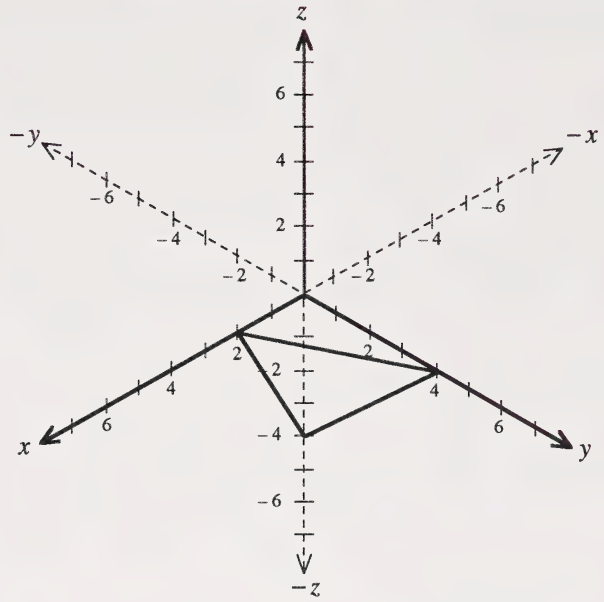
Let $x=0$, $z=0$.

Then, $0+y-0=4$

$y=4$

The y-intercept is $(0,4,0)$.

Construct the graph.



Try the following questions. Three-dimensional grids are provided in **Appendix B**.

1. Graph $2x-3y+2z=6$.

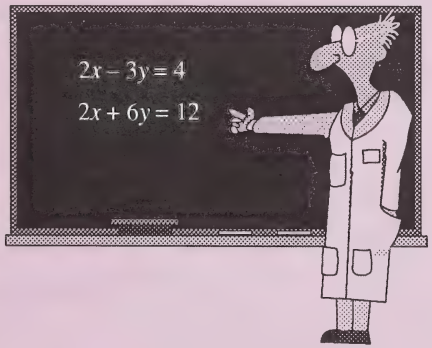
2. Graph $x+y-4z=4$.

For solutions to **Activity 2**, turn to **Appendix A**, **Topic 1**.

Topic 2 Solving Systems of Equations Algebraically

Introduction

In the last topic, systems of equations were solved graphically. This topic is going to show you the application of algebraic skills to solve systems of equations. You will investigate various methods of solutions and find out how these methods can be used to solve problems.



What Lies Ahead

Throughout this topic you will learn to

1. solve a system of two linear equations by comparison
2. solve a system of two linear equations by substitution
3. solve a system of two linear equations using addition or subtraction
4. solve a system of two linear equations using any algebraic method
5. solve word problems using any algebraic method

Now that you know what to expect, turn the page to begin your study of solving systems of equations algebraically.

Exploring Topic 2

Activity 1

Solve a system of two linear equations by comparison.

In the previous topic you were able to solve a system of linear equations by drawing graphs. This method is acceptable to use if the coordinates of the point of intersection are easy to read. However, sometimes it is impossible to read the exact values of the coordinates of the point of intersection. In this case you have to solve the system of equations algebraically.

There are many algebraic methods you can use. The first method is called the **comparison method**. The following example illustrates the principles of this method.

Example 1

Solve the following system of equations using the comparison method.

$2x - 3y = 4$ (1)

$2x + y = 12$ (2)

Since $2x$ is part of both equations, the comparison method will be useful.

Solution:

Perform algebraic operations to make the left side the same in each equation.

Isolate the common term in each equation.

From (1), add $3y$ to both sides and name the new equation (3).

$2x = 3y + 4$ (3)

From (2), subtract y from both sides, and name the new equation (4).

$2x = -y + 12$ (4)

Compare (3) and (4). The left sides are equal. Therefore, the right sides must also be equal.

Right side of (3) = Right side of (4)

$$3y + 4 = -y + 12$$

$3y + 4 + y = -y + 12 + y$ (Add y to both sides.)

$4y + 4 = 12$ (Simplify.)

$4y + 4 - 4 = 12 - 4$ (Subtract 4 from both sides.)

$4y = 8$ (Simplify.)

$\frac{4y}{4} = \frac{8}{4}$ (Divide both sides by 4.)

$y = 2$

Substitute $y = 2$ into (1). (You can substitute y into (1), (2), (3), or (4) and the solution will be the same.)

$$\begin{aligned} 2x - 3(2) &= 4 \\ 2x - 6 &= 4 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

Check (1).

LS	RS
$2x - 3y$	4
$2(5) - 3(2)$	4
$10 - 6$	4
4	4
LS =	RS

Check (2).

LS	RS
$2x + y$	12
$2(5) + 2$	12
$10 + 2$	12
12	12
LS =	RS

Therefore, $x = 5$ and $y = 2$ is the solution to this system of equations.

There are other ways to express this solution. The equivalent forms are as follows:

- The solution is $(x, y) = (5, 2)$.
- The coordinates of the point of intersection is (5, 2).

Now try some exercises. Complete either a or b in each question.

1. Solve the following system of equations by comparison.

   a. $2x - y = 4$ (1)
      $5x + y = 17$ (2)

   b. $2x + 3y = 20$ (1)
      $2x - 7y = -42$ (2)

2. Solve the following system of equations by comparison.

   a. $2x + 3y = 19$ (1)
      $2x - \frac{1}{3}y = -1$ (2)

   b. $\frac{1}{2}x - 5y = 1$ (1)
      $3x - 5y = 11$ (2)

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 2**.

Activity 2

Solve a system of two linear equations by substitution.

If you can isolate one of the variables on the left side, you can obtain an expression for this variable. You can substitute this expression into the other equation and get a new equation with only one variable. A linear equation in one variable would be easy to solve. Example 2 shows you this method called the substitution method.

Example 2

Solve the following system of equations.

$$x+3y=3 \quad (1)$$
$$\frac{1}{3}x-3y=-\frac{1}{3} \quad (2)$$

Solution:

Isolate x in equation (1).

From (1), subtract $3y$ from both sides and name the new equation (3).

$$x=-3y+3 \quad (3)$$

Substitute $-3y+3$ for x into (2).

$$\frac{1}{3}x-3y=-\frac{1}{3} \quad (2)$$
$$\frac{1}{3}(-3y+3)-3y=-\frac{1}{3} \quad \text{(Use the distributive property.)}$$
$$-y+1-3y=-\frac{1}{3}$$
$$-4y+1=-\frac{1}{3} \quad \text{(Multiply each term by 3.)}$$
$$-12y+3=-1$$
$$-12y=-1-3$$
$$-12y=-4$$
$$y=\frac{-4}{-12}$$
$$=\frac{1}{3}$$

Substitute $y=\frac{1}{3}$ into (3).

$$x=-3\left(\frac{1}{3}\right)+3$$
$$=-1+3$$
$$=2$$

Check (1).

LS	RS
$x+3y$	3
$(2)+3\left(\frac{1}{3}\right)$	3
$2+1$	3
3	3

LS = RS

Check (2).

LS	RS
$\frac{1}{3}x-3y$	$-\frac{1}{3}$
$\frac{1}{3}(2)-3\left(\frac{1}{3}\right)$	$-\frac{1}{3}$
$\frac{2}{3}-\frac{3}{3}$	$-\frac{1}{3}$
$-\frac{1}{3}$	$-\frac{1}{3}$

LS = RS

Thus, $x=2$ and $y=\frac{1}{3}$ is the solution.

Try the following question.

Do either a or b in the following questions.

1. Solve each of the following systems of equations by substitution.

   a. $x-5y=-25$ (1)
      $8x+7y=82$ (2)

   b. $5x+2y=29$ (1)
      $4x-y=5$ (2)

2. Solve each of the following systems of equations by substitution.

   a. $x+6y=6$ (1)
      $\frac{1}{4}x+3y=2$ (2)

   b. $3x-y=8\frac{4}{5}$ (1)
      $\frac{1}{6}x+\frac{5}{2}y=1$ (2)

For solutions to **Activity 2**, turn to **Appendix A**, **Topic 2**.

Activity 3

Solve a system of two linear equations using addition or subtraction.

Sometimes it is not easy to isolate a variable on one side. The substitution method is not always convenient for solving a system of equations. Instead you can use the **addition-subtraction method**. This method is also called the **elimination method**. Since you can add an equal amount to both sides of an equation or subtract an equal amount from both sides of an equation, it is possible to add or subtract two equations. The sum or difference on each side should still be equal. If the coefficients of the two terms of the same variable are equal, you can eliminate one of the variables by adding or subtracting two equations. This method is shown in the following example.

Example 3

Solve the following system of equations.

$2x+7y=9$ (1)
$5x-2y=3$ (2)

Solution:

Try to elimate the x-terms.

If you multiply (1) by 5, the first term becomes $10x$.

If you multiply (2) by 2, its first term becomes $10x$.

See how it is done.

$$\begin{array}{lll} (1)\times 5: & 10x+35y=45 & (3) \\ (2)\times 2: & 10x-4y=6 & (4) \\ (3)-(4): & 0x+39y=39 & \\ & 39y=39 & \\ & y=1 & \end{array}$$

Substitute $y=1$ into (1).

$$\begin{array}{ll} 2x+7(1)=9 & \text{(Subtract 7 from both sides.)} \\ 2x+7=9 & \\ 2x+7-7=9-7 & \text{(Divide both sides by 2.)} \\ 2x=2 & \\ x=1 & \end{array}$$

If $x=y$ and $u=v$, then $x-u=y-v$.

10 is the L.C.M. of 2 and 5.

If you multiply both sides of the equation by the same number, the equality does not change.

(3) − (4):

$$\begin{aligned} 10x-10x &= 0x \\ &= 0 \end{aligned}$$

$$\begin{aligned} 35y-(-4y) &= 35y+4y \\ &= 39y \end{aligned}$$

$$45-6=39$$

Check (1).

LS	RS
$2x+7y$	9
$2(1)+7(1)$	9
$2+7$	9
9	9
LS =	RS

Check (2).

LS	RS
$5x-2y$	3
$5(1)-2(1)$	3
$5-2$	3
3	3
LS =	RS

Thus, the solution is $x=1$ and $y=1$.

In Example 3, the subtraction method is used to eliminate the x-terms. Can the y-terms be eliminated? Look at Example 4.

Example 4

Solve the following system of equations.

$2x+7y=9$ (1)
$5x-2y=3$ (2)

Solution:

The coefficients of the two y-terms are 7 and -2. The L.C.M. of 7 and 2 is 14.

(1) $\times 2$: $4x+14y=18$ (3)
(2) $\times 7$: $35x-14y=21$ (4)

Since the y-term of (3) is positive and the y-term of (4) is negative, you have to use the addition method.

(3) + (4): $39x+0y=39$
$38x=35$
$x=1$

Substitute $x=1$ into (1).

$2(1)+7y=9$
$2+7y=9$
$2+7y-2=9-2$
$7y=7$
$y=1$

The solution is $x=1$ and $y=1$.

(3) + (4):
$4x+35x=39x$
$14y+(-14y)=0y$
$=0$
$18+21=39$

The answer for Example 4 has been checked. See Example 3.

If rational coefficients are involved, multiply every term by a common denominator first. The following example shows you the procedures.

Example 5

Solve the following system of equations.

$$3x-\frac{1}{2}y=6 \quad (1)$$

$$2x+\frac{1}{5}y=4 \quad (2)$$

Solution:

Eliminate the denominators.

$(1)\times 2$: $6x-y=12$ (3) (Multiply every term by 2.)

$(2)\times 5$: $10x+y=20$ (4) (Multiply every term by 5.)

You can see that the $-y$ and $+y$ will cancel out if you use the addition method.

$(3)+(4)$: $16x+0y=32$

$16x=32$

$x=2$

Substitute $x=2$ in (4).

$10(2)+y=20$

$20+y=20$

$y=0$

Check (1).

LS	RS
$3x-\frac{1}{2}y$	6
$3(2)-\frac{1}{2}(0)$	6
6	6
LS =	RS

Check (2).

LS	RS
$2x+\frac{1}{5}y$	4
$2(2)+\frac{1}{5}(0)$	4
4	4
LS =	RS

Thus, $x=2$ and $y=0$ is the solution.

The new system of (3) and (4) is equivalent to the original system of equations.

Example 6

Solve following system of equations.

$x - 0.2y = 1.5$ (1)

$0.5x - y = -0.15$ (2)

Solution:

Multiply (2) by a number that will make a coefficient in (2) equal to the corresponding coefficient in (1).

(2) $\times 2$: $1x - 2y = -0.3$

$x - 2y = -0.3$ (3)

Since the first term of (1) equals the first term of (3), they can be eliminated by the subtraction method.

(1) $-$ (3): $1.8y = 1.8$

$y = 1$

Substitute $y = 1$ in (1).

$x - 0.2(1) = 1.5$

$x = 1.5 + 0.2$

$= 1.7$

Check (1).

LS	RS
$x - 0.2y$	1.5
$1.7 - 0.2(1)$	1.5
1.5	1.5
LS =	RS

Check (2).

LS	RS
$0.5x - y$	-0.15
$0.5(1.7) - 1$	-0.15
$0.85 - 1$	-0.15
-0.15	-0.15
LS =	RS

Thus, $x = 1.7$ and $y = 1$ is the solution.

$-0.2y - (-2y) = -0.2y + 2y$

$= 1.8y$

$1.5 - (-0.3) = 1.5 + 0.3$

$= 1.8$

Now do the following questions. Do either a or b in each.

1. Use the addition-subtraction method to solve the following systems of equations.

   a. $2x-3y=4$ (1)
      $7x-2y=31$ (2)

   b. $5x-2y=7$ (1)
      $7x-3y=9$ (2)

2. Use the addition-subtraction method to solve the following systems of equations.

   a. $3x-5y=7$ (1)
      $6x+7y=31$ (2)

   b. $2x+y=16$ (1)
      $3x-2y=3$ (2)

3. Use the elimination method to solve the following systems of equations.

   a. $\frac{1}{2}x+3y=31$ (1)
      $\frac{1}{4}x+\frac{1}{3}y=5$ (2)

   b. $\frac{1}{3}x-2y=-4$ (1)
      $\frac{1}{2}x+\frac{1}{3}y=4$ (2)

4. Use the elimination method to solve the following systems of equations.

   a. $0.2x+3y=6.3$ (1)
      $0.3x+2y=4.45$ (2)

   b. $0.2x+0.7y=5$ (1)
      $3x-0.5y=9$ (2)

For solutions to **Activity 3**, turn to **Appendix A**, **Topic 2**.

Activity 4

Solve a system of two linear equations using any algebraic method.

You have now looked at three different algebraic methods to solve a linear system. Note that any of the three methods could be used to solve any linear system. However, for a particular question, one of the methods may be easier to apply. Look at some situations to illustrate this point.

Look at the following system.

$x+3y=17$ (1)
$5x-4y=27$ (2)

Since this system is quite easy to isolate x in equation (1), the **substitution method** is the easiest way.

$x=17-3y$ (1)

Substitute $17-3y$ for the x in (2) and solve for the y-value.

Look at the following system.

$3y=2x+3$ (1)
$3y=-x+5$ (2)

Since the y-terms in both equations are equal, the **comparison method** is the easiest way Since $3y=3y$, you can also say $2x+3=-x+5$; then solve for x.

Look at the following system.

$2x-4y=17$ (1)
$3x+2y=3$ (2)

For this system the **addition-subtraction (elimination) method** is the easiest way.

The substitution method would create some additional concerns because in order to isolate one variable, you would create fractions (which could be a source of potential problems).

Consider the following:

$2x=17-4y$ (1)

$x=\dfrac{17+4y}{2}$ (Divide by 2.)

As you can see, a fraction results when x is isolated. Often it is best to stay away from fractions if possible. Involving fractions could make calculations more complicated. It is advantageous to keep all calculations as simple as possible.

In this example it is probably easier to eliminate the y-value. Multiply (2) by 2 and you will get $4y$. Add the two equations and the $-4y$ will eliminate the $4y$. Then you can easily solve for the x-value.

Example 7

Use all three methods to solve the following system of equations.

$5x-6y=21$ (1)

$3x+2y=35$ (2)

Solution:

Substitution Method

From (2), $2y=35-3x$

$$y=\frac{35-3x}{2}$$

Substitute $y=\frac{35-3x}{2}$ into (1).

$$5x-6\left(\frac{35-3x}{2}\right)=21$$

$$5x-3(35-3x)=21$$

$$5x-105+9x=21$$

$$14x-105=21$$

$$14x=105+21$$

$$14x=126$$

$$x=9$$

Substitute $x=9$ into (2).

$$3(9)+2y=35$$

$$27+2y=35$$

$$2y=8$$

$$y=4$$

Comparison Method

From (1), $5x=6y+21$

$$x=\frac{6y+21}{5} \quad (3)$$

From (2), $3x=35-2y$

$$x=\frac{35-2y}{3} \quad (4)$$

LS of (3) = LS of (4)

RS of (3) = RS of (4)

Compare ③ and ④.

$$\frac{6y+21}{5}=\frac{35-2y}{3}$$

$$(15)\left(\frac{6y+21}{5}\right)=\left(\frac{35-2y}{3}\right)(15) \quad \text{(Multiply both sides by 15.)}$$

$$3(6y+21)=5(35-2y)$$
$$18y+63=175-10y$$
$$18y+10y=175-63$$
$$28y=112$$
$$y=\frac{112}{28}$$
$$=4$$

Substitute $y=4$ into ①.

$$5x-6(4)=21$$
$$5x-24=21$$
$$5x=45$$
$$x=9$$

Elimination Method

Eliminate y.

$$3x+2y=35 \quad ②$$
$$②\times 3: \quad 9x+6y=105 \quad ③$$
$$5x-6y=21 \quad ①$$
$$①+③: \quad 14x+0y=126$$
$$14x=126$$
$$x=\frac{126}{14}$$
$$=9$$

Substitute $x=9$ into ②.

$$3(9)+2y=35$$
$$27+2y=35$$
$$2y=35-27$$
$$2y=8$$
$$y=4$$

Check (1).

LS	RS
$5x-6y$	21
$5(9)-6(4)$	21
$45-24$	21
21	21
LS =	RS

Check (2).

LS	RS
$3x+2y$	35
$3(9)+2(4)$	35
$27+8$	35
35	35
LS =	RS

Thus, $x=9$ and $y=4$ is the solution.

If you were asked to solve Example 7, which method would you prefer?

Answer any three questions.

For each of the following questions, which one of the three methods (substitution, comparison, elimination) is easier to apply? Explain. **(Do not solve.)**

1. $3x-y=8$ (1)
   $5x-6y=15$ (2)

2. $x-3y=7$ (1)
   $4x-9y=28$ (2)

3. $5x-3y=19$ (1)
   $5x+7y=3$ (2)

4. $3x-7y=11$ (1)
   $6x+3y=8$ (2)

5. $11x-3y=21$ (1)
   $5x+7y=18$ (2)

For solutions to **Activity 4**, turn to **Appendix A**, **Topic 2**.

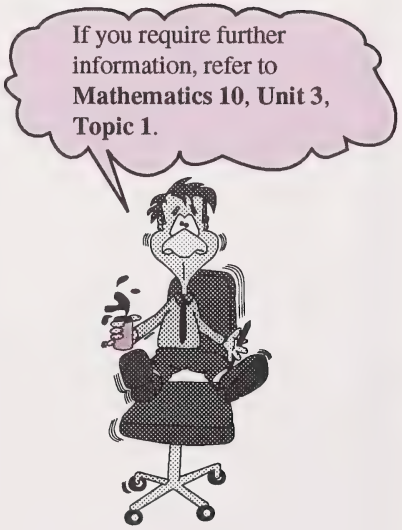
Activity 5

Solve word problems using any algebraic method.

Before you solve any word problem, you have to translate the facts of the problem into algebraic expressions. If the question has two unknowns, then two different variables would be involved in the algebraic expressions. The relation between the variables can be used to determine an equation.

Verbal Expression	Equation
A table and four chairs cost \$150.00.	$x+4y=150$
Five years ago Gynette was seven times her son's age.	$(x-5)=7(y-5)$
Two times one number exceeds another number by seven.	$2x=y+7$
The length of a field is two metres less than three times its width.	$l=3w-2$

If two variables are involved in a problem, then two equations must be determined, and they form a system of equations. The solutions to the system of equations is the answer to the problem. Look at the following example.



Example 8

One number exceeds the other by three. The sum of the two numbers is 53. Find the two numbers.

Solution:

Let x be the smaller number and y the larger number.

The equations are as follows:

$y = x + 3$ (1) (One number exceeds the other by three.)

$x + y = 53$ (2) (The sum of the two numbers is 53.)

Substitute $x + 3$ for y in (2).

$$x + (x + 3) = 53$$
$$2x + 3 = 53 \quad \text{(Subtract 3 from both sides.)}$$
$$2x = 50 \quad \text{(Divide both sides by 2.)}$$
$$x = 25$$

Substitute 25 for x in (1).

$$y = x + 3$$
$$= 25 + 3$$
$$= 28$$

Check (1).

LS	=	RS
y		$x+3$
28		$25+3$
28		28
LS	=	RS

Check (2).

LS	=	RS
$x+y$		53
$25+28$		53
53		53
LS	=	RS

Thus, the two numbers are 25 and 28.

There are many different types of word problems. It is not possible to show you all of them. The next four examples represent four different types of problems.

To solve word problem, do the following:

- Understand the problem.
- Develop a plan. Derive equations from the facts.
- Carry out the plan. Solve the system of equations.
- Look back and verify the solution.

Example 9

It cost \$27 for 1 kg of ground beef and 3 kg of steak. Three months ago, the price of ground beef was 10% cheaper and the price of steak was 15% cheaper. The cost of the same order was \$23.10. What is the present cost of a kilogram of both ground beef and steak?

Solution:

Let x be the cost of a kilogram of ground beef and y the cost of a kilogram of steak.

$$x+3y=27 \quad (1)$$
$$0.90x+0.85(3y)=23.1$$
$$0.90x+2.55y=23.1 \quad (2)$$

$(2)\times 10$: $9x+25.5y=231$ (3)

$(1)\times 9$: $9x+27y=243$ (4)

$$-9x+25.5y=231$$

$(4)-(3)$: $1.5y=12$

$$y=\frac{12}{1.5}$$
$$=8$$

Substitute y into (1).

$$x+3(8)=27$$
$$x+24=27$$
$$x=3$$

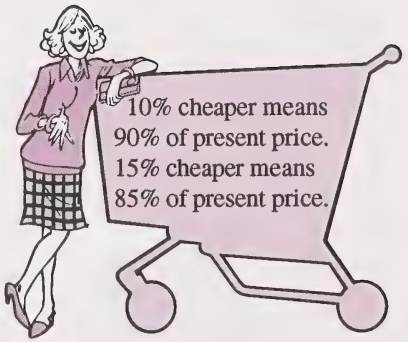
Check (1).

LS		RS
$x+3y$		27
$3+(3)8$		27
$3+24$		27
27		27
LS	=	RS

Check (2).

LS		RS
$0.9x+2.55y$		23.1
$0.9(3)+2.55(8)$		23.1
$2.7+20.4$		23.1
23.1		23.1
LS	=	RS

Thus, ground beef costs \$3/kg and steak costs \$8/kg.



Note: The cost of 3 kg of steak is $3y$. The cost of 3 kg of steak after a 15% discount is $0.85(3y)$. The cost of 1 kg of ground beef after a 10% discount is $0.90x$.

Example 10

Marvin has \$445 annual income from his investments bearing 8% and $9\frac{1}{2}\%$ interest. If the total investment was \$5000, how much did he invest at each rate?

Solution:

Let x be the investment at 8% and y the investment at $9\frac{1}{2}\%$.

$x + y = 5000$ (1) (total investment)

$\frac{8}{100}x + \frac{9.5}{100}y = 445$ (2) (interest earned)

From (2), $0.08x + 0.095y = 445$.

(2) $\times 100$: $8x + 9.5y = 44\,500$ (3)

(1) $\times 8$: $-8x + 8y = 40\,000$ (4)

(3) $-$ (4): $1.5y = 4500$

$$y = \frac{4500}{1.5}$$
$$= \$3000$$

Substitute $y = 3000$ into (1).

$$x + 3000 = 5000$$
$$x = 5000 - 3000$$
$$= 2000$$

Check (1).

LS		RS
$x + y$		5000
$2000 + 3000$		5000
5000		5000
LS	=	RS

Check (2).

LS		RS
$0.08x + .095y$		445
$0.08(2000) + .095(3000)$		445
$160 + 285$		445
445		445
LS	=	RS

Thus, Marvin invested \$3000.00 at $9\frac{1}{2}\%$ and \$2000.00 at 8%.

Multiply (2) by 100 in order to change the coefficient of the x-term to a whole number.

Example 11

There is a two-digit number. The sum of the two digits is eight. Four times the tens digit is seventeen larger than the units digit. Find this two-digit number.

Solution:

Let x be the tens digit and y the units digit.

$$x+y=8 \quad (1)$$
$$4x=y+17 \quad (2)$$

From (2), $4x-y=17$. (3) (Subtract y from both sides.)

$$\begin{array}{rl} & x+y=8 \quad (1) \\ & +\ 4x-y=17 \quad (3) \\ \hline (1)+(3): & 5x+0y=25 \\ & x=5 \end{array}$$

Substitute $x=5$ into (1).

$$\begin{aligned} x+y&=8 \\ 5+y&=8 \\ y&=8-5 \\ &=3 \end{aligned}$$

Check (1).

LS		RS
$x+y$		8
$5+3$		8
8		8
LS	=	RS

Check (2).

LS		RS
$4x$		$y+17$
$4(5)$		$(3)+17$
20		20
LS	=	RS

The tens digit is 5 and the units digit is 3. Thus, the two-digit number is 53.

Example 12

It takes George $2\frac{1}{2}$ h to row his boat 6 km upstream and $1\frac{1}{2}$ h to return. What is the speed of the boat in still water, and what is the speed of the current?

Solution:

Let x be the speed of the boat in still water and y the speed of the current.
distance = speed × time

Upstream: $6 \text{ km} = (x-y)\times 2\frac{1}{2}$ (1)

Downstream: $6 \text{ km} = (x+y)\times 1\frac{1}{2}$ (2)

From (1), $2.5x - 2.5y = 6$.

(1) × 30: $75x - 75y = 180$ (3)

From (2), $1.5x + 1.5y = 6$.

(2) × 50: $75x + 75y = 300$ (4)

$$\begin{array}{rl} & 75x - 75y = 180 \\ \hline (3)+(4): & 150x + 0y = 480 \end{array}$$

$$x = \frac{480}{150}$$
$$= 3.2 \text{ km/h}$$

Substitute $x = 3.2$ km/h into (1).

$$6 = (3.2 - y)\times 2.5$$
$$6 = 8 - 2.5y$$
$$2.5y = 8 - 6$$
$$2.5y = 2$$
$$y = \frac{2}{2.5}$$
$$= 0.8 \text{ km/h}$$

Check (1).

LS	RS
6	$(x-y)\times 2.5$
6	$(3.2-0.8)\times 2.5$
6	2.4×2.5
6	6

LS = RS

Check (2).

LS	RS
6	$(x+y)\times 1.5$
6	$(3.2+0.8)\times 1.5$
6	4×1.5
6	6

LS = RS

The speed of the boat in still water is 3.2 km/h.
The speed of the current is 0.8 km/h.

When rowing upstream, the actual speed is equal to the speed of boat in still water minus the speed of the current $(x-y)$. When rowing downstream, the actual speed is equal to the speed of boat in still water plus the speed of the current $(x+y)$.

Try the following questions.

Do all of question 1 and do the even or odd-numbered questions for questions 2 to 11.

1. Translate the following verbal expressions into algebraic expressions.

   a. One can of tomato soup and four cans of orange juice cost $7.90.

   b. A box contains $6.00 in dimes and quarters.

   c. Two times the difference of two numbers is seven less than their sum.

2. One number is three less than the other. The sum of these two numbers is 99. Find the two numbers.

3. The difference of two numbers is 24. Two times the smaller number exceeds the larger number by seventeen. Find the two numbers.

4. It costs $35.20 for 3 kg of turkey and 4 kg of club steak. Next month the price of turkey will advance 10% and the price of steak will advance 20%. The cost of the same order will be $41.52. What is the present cost of a kilogram of each?

5. It costs $7.60 for 4 L of oil and 2 L of soft drink. There will be a sale next week. The price of oil will be 5% cheaper and the price of the soft drink will be 20% cheaper. The cost of the same order will be $6.80. What is the present cost of a litre of each.

6. Whitley earns $235 annual interest from her investments bearing 7% and 11% interest. If her total investment was $2500, how much did she invest at each rate?

7. Denzil invested $3000, part at 9% per annum and the remainder at 12%. One year later, the interest earned on the 12% investment was $150 more than the interest earned on the 9% investment. How much did he invest at each rate?

8. There is a two-digit number. The sum of the two digits is seven. Three times the units digit is one larger than two times the tens digit. Find the two-digit number.

9. There is a two-digit number. The tens digit exceeds the units digit by five. This two-digit number is equal to ten times the sum of the two digits. Find the two-digit number.

10. It takes three hours to row a boat 9 km upstream and two hours to return. What is the speed of the boat in still water and what is the speed of the current?

11. The distance from Edmonton to Calgary is 295 km. The speed limit is 100 km/h on part of Highway 2 and 110 km/h on the rest of Highway 2. It took Peter $2\frac{3}{4}$ hours to drive from Edmonton to Calgary. How far did he drive at 100 km/h?

For solutions to **Activity 5**, turn to **Appendix A**, **Topic 2**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

Do either **Part A** or **Part B**.

Part A

If you had trouble with Example 10, try the following questions. Complete every step. Check every step with the solution provided in **Appendix A**.

1. Steve earned \$640.00 from one year investment certificates bearing 10% and 12% interest. His total investment was \$6000.00. How much did he invest at each rate?

Solution:

Let x equal the investment at 10% and y the investment at 12%.

Step 1: Change the percentages to decimal numbers.

10% is equal to _____.

12% is equal to _____.

Step 2: The interest earned at 10% is $x \times$ _____.

The interest earned at 12% is $y \times$ _____.

Step 3: Therefore, the total interest is

$0.10x + 0.12y = 640$. (1)

Therefore, the total investment is

$x + y = 6000$. (2)

Step 4: Since (1) has decimal coefficients, multiply (1) by 100 so that all the decimal coefficients can be changed to whole number coefficients.

(1) $\times 100$: _____ + _____ $= 64\,000$ (3)

Step 5: In order to make the coefficient of x in (2) and (3) equal, you have to multiply (2) by 10.

$$\begin{array}{rrll} & 10x + 12y = 64\,000 & (3) \\ (2) \times 10: & 10x + 10y = 60\,000 & (4) \\ (3) - (4): & 0x + 2y = 4000 & \text{(Elimate the } x\text{-terms.)} \\ & 2y = 4000 & \\ & y = 2000 & \end{array}$$

Step 6: Substitute $y = 2000$ into (2).

$x + 2000 = 6000$

$x =$ _____ $-$ _____

$=$ _____

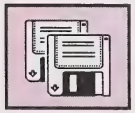
The investment at 10% is _____, and the

investment at 12% is _____.

2. Karen earns \$110 annually from investments bearing 6% and 8%. Her total investment was \$1500. How much did she invest at each rate.

For solutions to **Extra Help**, turn to **Appendix A, Topic 2.**

Part B



For extra help with solving systems of equations by graphing, substitution, and elimination , use the computer program titled, *Problem Solving in Algebra*[1], lessons 12, 13, and 15.

For extra help with the elimination method, use the computer program titled, *Algebra Drill and Practice II*[2], disk 1, part 4.

[1] Problem Solving in Algebra is a title of *Encyclopedia Britannica Publications Ltd.*

[2] Algebra Drill and Practice II is a title of *Conduit.*

Extensions

In **Topic 2**, all the examples and questions (except the first objective) are independent systems of equations. Can a dependent or inconsistent system of equations be solved algebraically? Look at the following examples.

Example 13

Solve the following system of equations.

$$x - 3y = 4 \quad (1)$$
$$2x - 6y = 1 \quad (2)$$

Solution:

$$(1) \times 2: \quad 2x - 6y = 8 \quad (3)$$
$$2x - 6y = 1 \quad (2)$$
$$(3) - (2): \quad 0 - 0 = 7$$
$$0 = 7$$

Since $0 \neq 7$, there is no solution because this system of equations is inconsistent.

Example 14

Solve the following system of equations.

$x-2y=3$ (1)
$2x-4y=6$ (2)

Solution:

(1) $\times 2$: $2x-4y=6$ (3)

Since (2) and (3) are the same, (2) $-$ (3) gives $0-0=0$ or $0=0$.

This is always true. In other words x and y can be any number as long as they maintain the same relation $x-2y=3$. This is because the system is dependent. Therefore, the solution is the line represented by $x-2y=3$.

You have learned how to solve systems of equations in two variables. Can you solve a system of equations in three or more unknowns?

Colin Maclaurin, a Scottish prodigy, developed a systematic method to solve systems of equations in three or more variables. He entered the University of Glasgow at age 11. At age 19, he became a professor of mathematics at Marischal College, Aberdeen. In 1725, on Sir Isaac Newton's recommendation, he was made professor of mathematics at the University of Edinburgh. His constributions to mathematics included extensions of Newton's work in calculus, geometry, and gravitation. He died at the age of 48.

Example 15

Solve a system of equations in three variables.

$x+y-z=2$ (1)
$x-2y+3z=4$ (2)
$x+y+2z=5$ (3)

Solution:

To solve this system of equations, you have to eliminate one of the variables from two different pairs of equations by using the elimination method.

Use (1) and (2) to eliminate x.

(1) $-$ (2): $3y-4z=-2$ (4)

Use (2) and (3) to eliminate x.

$(2)-(3)$: $-3y+z=-1$ (5)

You now have a new system of equations, (4) and (5), that has only two variables.

$(4)+(5)$: $-3z=-3$

$z=1$

Substitute $z=1$ into (4).

$3y-4(1)=-2$

$3y=2$

$y=\frac{2}{3}$

Substitute $y=\frac{2}{3}$ and $z=1$ into (1).

$x+\frac{2}{3}-1=2$

$x=2+1-\frac{2}{3}$

$=2\frac{1}{3}$

Therefore, in the system of equations $x=2\frac{1}{3}$, $y=\frac{2}{3}$, and $z=1$.

Do the following questions.

Solve the following questions algebraically.

1. $2x-3y=7$ (1)
   $4x-6y=8$ (2)

2. $3x-5y=6$ (1)
   $6x-10y=12$ (2)

3. $x-y+z=7$ (1)
   $2x+y-3z=5$ (2)
   $x-2y+z=10$ (3)

For solutions to **Extensions**, turn to **Appendix A**, **Topic 2**.

Unit Summary

What You Have Learned

In this unit you learned the following:

- Linear equations with two variables can be solved by using two equations that form a system of equations.
- Systems of equations can be solved by graphing.
- If the two graphs intersect at one point, the system is independent or intersecting.
- If the two graphs are parallel to each other, the system is inconsistent or parallel.
- If the two graphs coincide or are the same line, the system is dependent or coincident.
- Systems can be solved algebraically using one of the following methods: substitution, comparison, or addition or subtraction (elimination).
- Systems of equations can be used to solve written problems after the facts are appropriately arranged in two equations using two variables.

You are now ready to

complete the **Unit Assignment**.

Appendices

Appendix A
Solutions

Review

Topic 1 Solving Systems of Equations Graphically

Topic 2 Solving Systems of Equations Algebraically

Appendix B
Grids

Grids

Three-dimensional Grids

Appendix A
Solutions

Review

1. $3x-4=23$

$3x-4+4=23+4$

$3x=27$

$\frac{3x}{3}=\frac{27}{3}$

$x=9$

Check:

LS	RS
$3x-4$	23
$3(9)-4$	23
$27-4$	23
23	23
LS =	RS

2. $2(3x-5)=32$

$6x-10=32$

$6x-10+10=32+10$

$6x=42$

$\frac{6x}{6}=\frac{42}{6}$

$x=7$

Check:

LS	RS
$2(3x-5)$	32
$2[3(7)-5]$	32
$2(21-5)$	32
$2(16)$	32
32	32
LS =	RS

3. $4(x-3)+9x=-38$

$4x-12+9x=-38$

$13x-12=-38$

$13x-12+12=-38+12$

$13x=-26$

$\frac{13x}{13}=\frac{-26}{13}$

$x=-2$

Check:

LS	RS
$4(x-3)+9x$	-38
$4[(-2)-3]+9(-2)$	-38
$4(-5)-18$	-38
$-20-18$	-38
-38	-38
LS =	RS

4. $7m+2=5m+18$

$7m+2-2=5m+18-2$

$7m=5m+16$

$7m-5m=5m+16-5m$

$2m=16$

$\frac{2m}{2}=\frac{16}{2}$

$m=8$

Check:	LS	RS
	$7m+2$	$5m+18$
	$7(8)+2$	$5(8)+18$
	$56+2$	$40+18$
	58	58
	LS =	RS

5. $3(2r+4)=4(5r-4)$

$6r+12=20r-16$

$6r+12-12=20r-16-12$

$6r=20r-28$

$6r-20r=20r-28-20r$

$-14r=-28$

$\frac{-14r}{-14}=\frac{-28}{-14}$

$r=2$

Check:	LS	RS
	$3(2r+4)$	$4(5r-4)$
	$3[2(2)+4]$	$4[5(2)-4]$
	$3(4+4)$	$4(10-4)$
	$3(8)$	$4(6)$
	24	24
	LS =	RS

6. $x-5=8-2(x+2)$

$x-5=8-2x-4$

$x-5=4-2x$

$x-5+5=4-2x+5$

$x=-2x+9$

$x+2x=-2x+9+2x$

$3x=9$

$\frac{3x}{3}=\frac{9}{3}$

$x=3$

Check:	LS	RS
	$x-5$	$8-2(x+2)$
	$(3)-5$	$8-2[(3)+2]$
	-2	$8-2(5)$
	-2	$8-10$
	-2	-2
	LS =	RS

7. $2(5x-11)-6=3(x-7)-15$

$10x-22-6=3x-21-15$

$10x-28=3x-36$

$10x-28+28=3x-36+28$

$10x=3x-8$

$10x-3x=3x-8-3x$

$7x=-8$

$\frac{7x}{7}=\frac{-8}{7}$

$x=\frac{-8}{7}$

Check:

LS	RS
$2(5x-11)-6$	$3(x-7)-15$
$2\left[5\left(\frac{-8}{7}\right)-11\right]-6$	$3\left[\left(\frac{-8}{7}\right)-7\right]-15$
$2\left(\frac{-40}{7}-11\right)-6$	$3\left(\frac{-57}{7}\right)-15$
$2\left(\frac{-117}{7}\right)-6$	$\frac{-171}{7}-15$
$\frac{-234}{7}-6$	$\frac{-171}{7}-\frac{105}{7}$
$\frac{-234}{7}-\frac{42}{7}$	$\frac{-276}{7}$
$\frac{-276}{7}$	$\frac{-276}{7}$

LS = RS

Exploring Topic 1

Activity 1

Solve and classify a system of two linear equations by graphing.

1. a. Make a table of values for each equation.

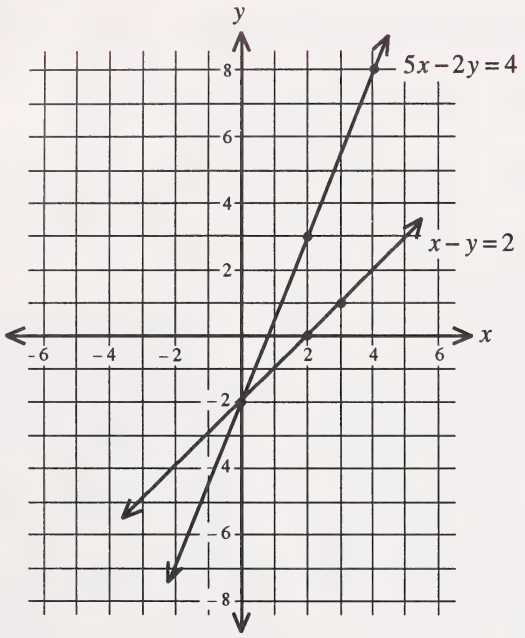
$5x-2y=4$

x	y
0	-2
2	3
4	8

$x-y=2$

x	y
0	-2
2	0
3	1

(Other values may be used in the table of values.)



The point of intersection is $(0, -2)$.

Check (1).

LS	RS
$5x-2y$	4
$5(0)-2(-2)$	4
4	4
LS =	RS

Check (2).

LS	RS
$x-y$	2
$0-(-2)$	2
2	2
LS =	RS

Therefore, $x = 0$ and $y = -2$, or the point of intersection is $(0, -2)$.

b. Make a table of values for each equation.

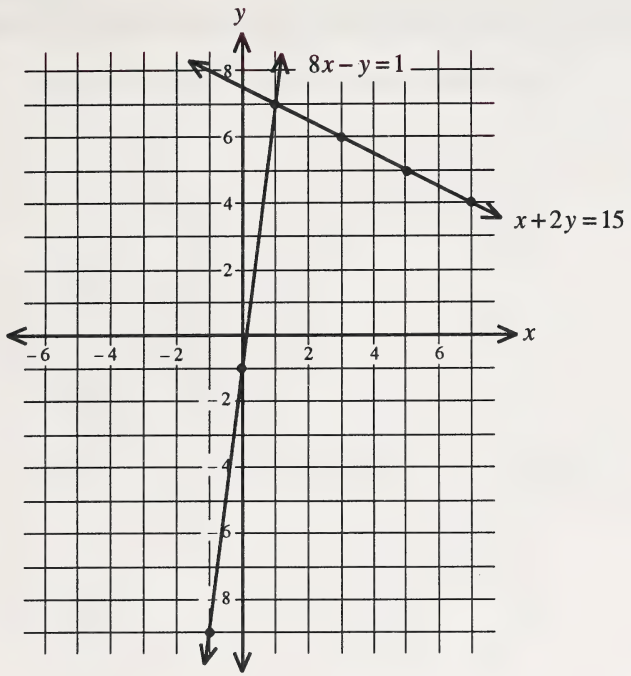
$8x - y = 1$

x	y
0	−1
1	7
−1	−9

$x + 2y = 15$

x	y
3	6
5	5
7	4

(Other values may be used in the table of values.)



The point of intersection is $(1, 7)$.

Check (1).

LS	RS
$8x - y$	1
$8(1) - 7$	1
$8 - 7$	1
1	1
LS =	RS

Check (2).

LS	RS
$x + 2y$	15
$1 + 2(7)$	15
$1 + 14$	15
15	15
LS =	RS

Thus, $x = 1$ and $y = 7$, or the point of intersection is $(1, 7)$.

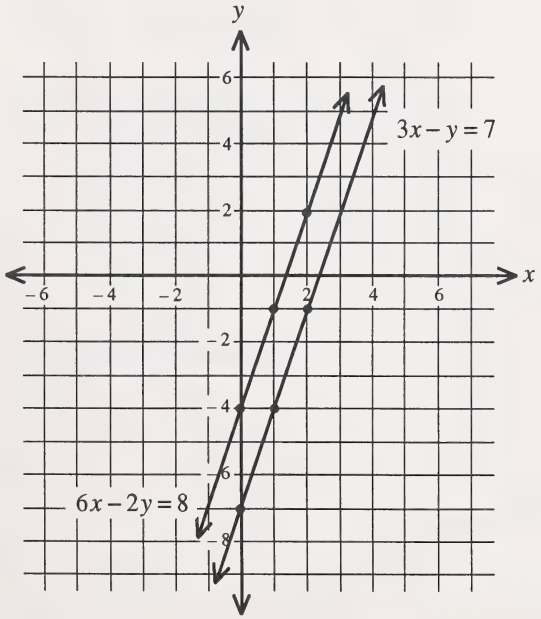
2. a. Make a table of values for each equation.

$3x - y = 7$

x	y
0	−7
1	−4
2	−1

$6x - 2y = 8$

x	y
0	−4
1	−1
2	2



The two lines are parallel; thus, there is no solution.

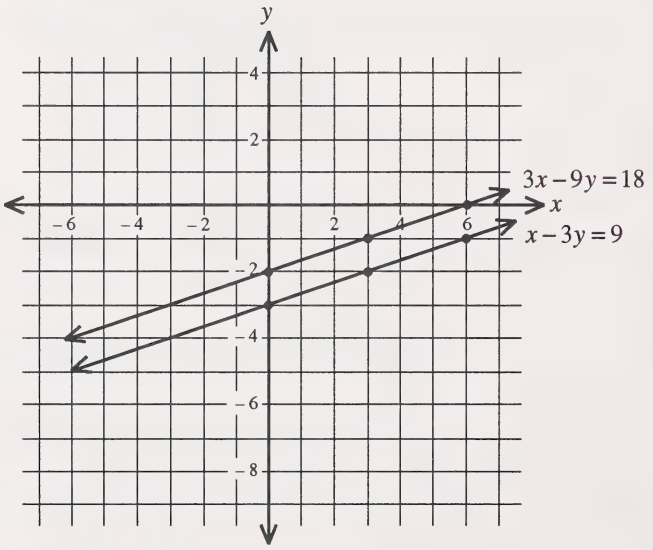
b. Make a table of values for each equation.

$x - 3y = 9$

x	y
0	−3
3	−2
6	−1

$3x - 9y = 18$

x	y
0	−2
3	−1
6	0



The two lines are parallel; thus, there is no solution.

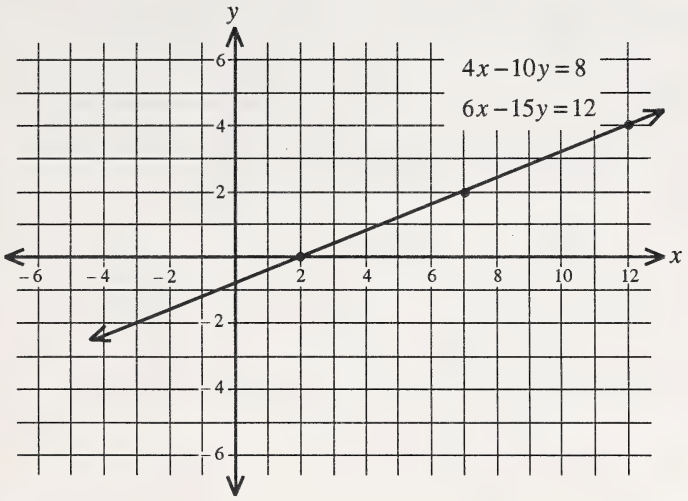
3. a. Make a table of values for each equation.

$4x-10y=8$

x	y
2	0
7	2
12	4

$6x-15y=12$

x	y
2	0
7	2
12	4



The two lines coincide; thus, there are an infinite number of solutions.

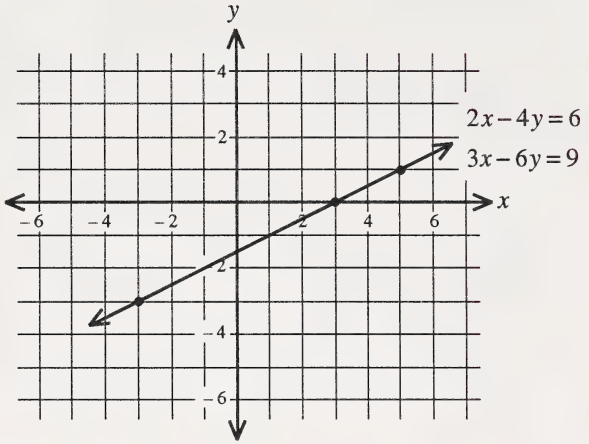
b. Make a table of values for each equation.

$2x-4y=6$

x	y
3	0
5	1
−3	−3

$3x-6y=9$

x	y
3	0
5	1
−3	−3



The two lines coincide. Thus, there are an infinite number of solutions.

Notice that if you multiply each term of (1) by $\frac{3}{2}$, you get (2). In questions 3a and 3b, (2) is a multiple of (1).

4. a. ① $\times 2$: $2x-3y=14$ ③
② $\times 2$: $x-2y=7$ ④

① $\times 2$ means multiply each term of ① by 2.

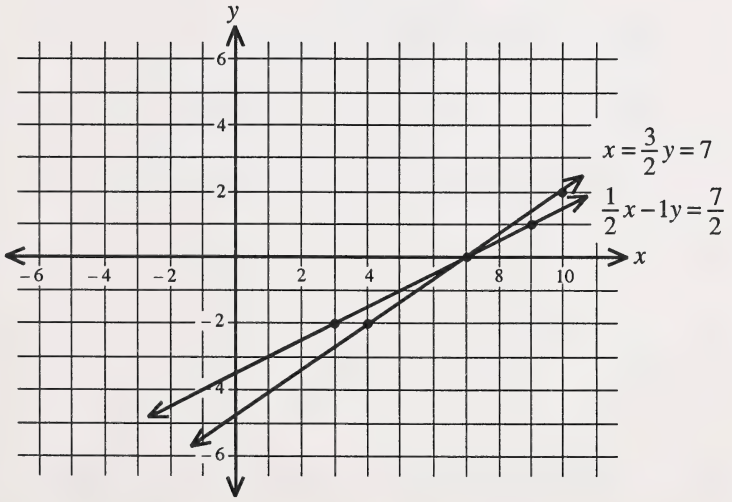
Make a table of values for equations ③ and ④.

$2x-3y=14$

x	y
7	0
10	2
4	−2

$x-2y=7$

x	y
7	0
9	1
3	−2



The point of intersection is $(7, 0)$. Thus, $x=7$ and $y=0$.

Check ①.

LS	RS
$x-\frac{3}{2}y$	7
$7-\frac{3}{2}(0)$	7
7	7

LS = RS

Check ②.

LS	RS
$\frac{1}{2}x-1y$	$\frac{7}{2}$
$\frac{1}{2}(7)-1(0)$	$\frac{7}{2}$
$\frac{7}{2}$	$\frac{7}{2}$

LS = RS

b. (1) $\times 3$: $3x+y=9$ (3)

(2) $\times 5$: $2x+5y=19$ (4)

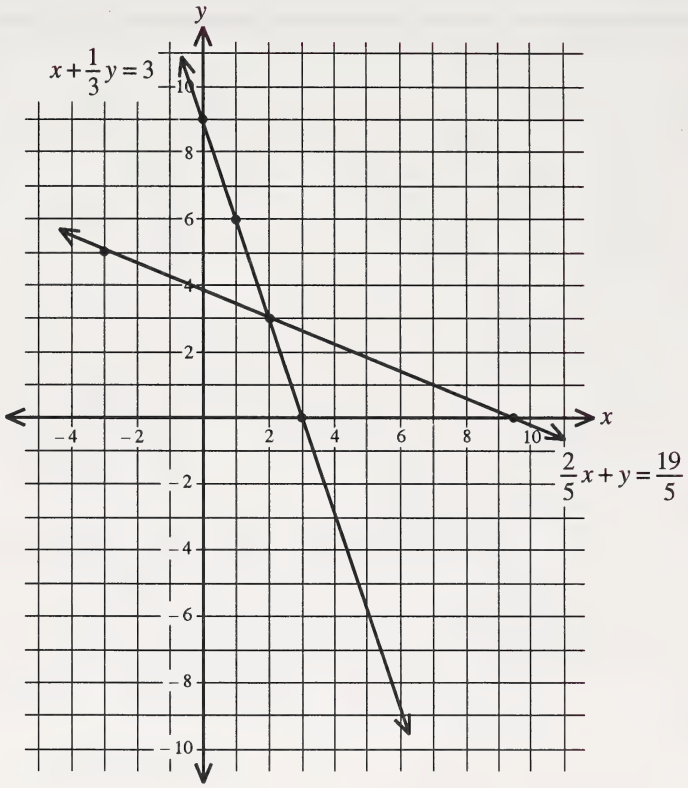
Make a table of values for (3) and (4).

$3x+y=9$

x	y
0	9
3	0
1	6

$2x+5y=19$

x	y
2	3
-3	5
9.5	0



The point of intersection is $(2, 3)$. Thus, $x=2$ and $y=3$.

Check (1).

LS	RS
$x+\frac{1}{3}y$	3
$2+\frac{1}{3}(3)$	3
$2+1$	3
3	3

LS = RS

Check (2).

LS	RS
$\frac{2}{5}x+y$	$\frac{19}{5}$
$\frac{2}{5}(2)+3$	$\frac{19}{5}$
$\frac{4}{5}+3$	$\frac{19}{5}$
$\frac{4}{5}+\frac{15}{5}$	$\frac{19}{5}$
$\frac{19}{5}$	$\frac{19}{5}$

LS = RS

5. Questions 1a and 1b are independent.
Questions 2a and 2b are inconcistent.
Questions 3a and 3b are dependent.

6. Let the two numbers be x and y.

$\frac{1}{3}(x+y)=1$ (1)

$3x+\frac{1}{2}y=4$ (2)

(1) $\times 3$: $x+y=3$ (3)

(2) $\times 2$: $6x+y=8$ (4)

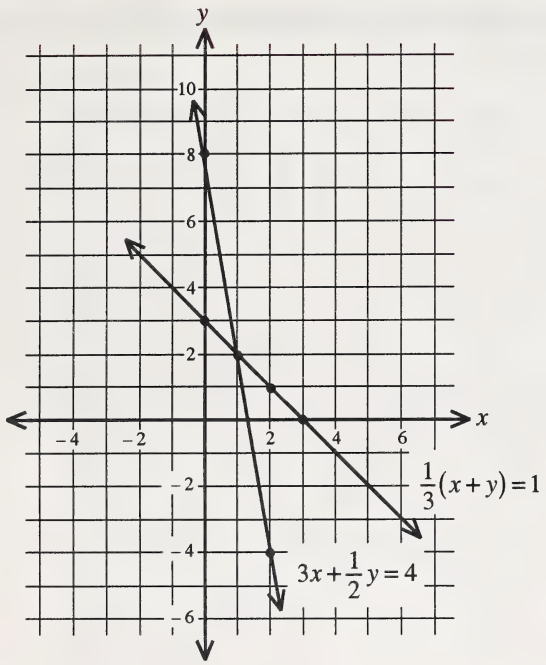
Make a table of values for (3) and (4).

$x+y=3$

x	y
0	3
3	0
2	1

$6x+y=8$

x	y
0	8
1	2
2	−4



The point of intersection is $(1, 2)$. Thus, the two numbers are 1 and 2.

Check (1).

LS	RS
$\frac{1}{3}(x+y)$	1
$\frac{1}{3}(1+2)$	1
$\frac{1}{3}(3)$	1
1	1

LS = RS

Check (2).

LS	RS
$3x+\frac{1}{2}y$	4
$3(1)+\frac{1}{2}(2)$	4
$3+1$	4
4	4

LS = RS

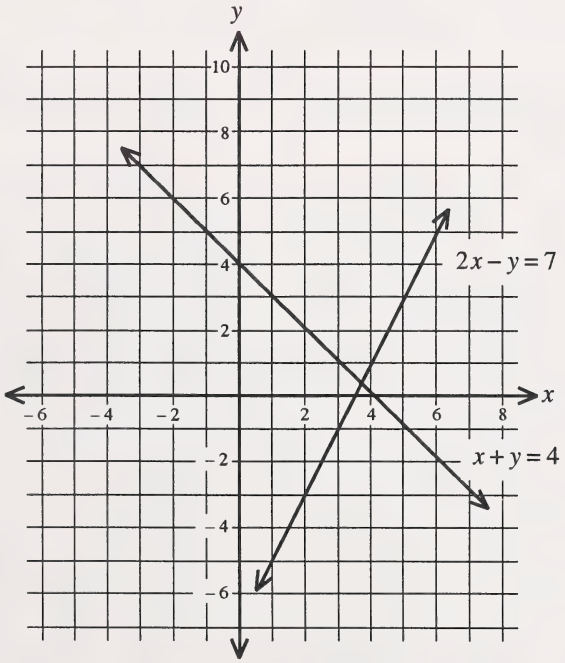
7. a. Change (1) to $y = mx + b$ form.

$-y = -2x + 7$

$y = 2x - 7$

Change (2) to $y = mx + b$ form.

$y = -x + 4$



The equations are independent or intersecting.

b. Change (1) to $y = mx + b$ form.

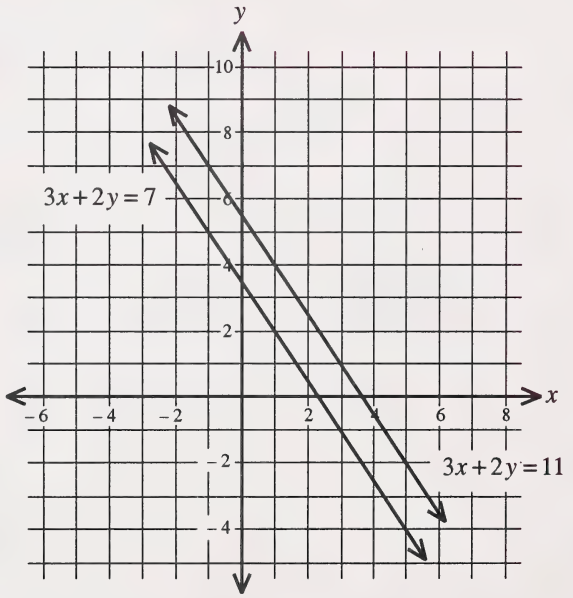
$2y = -3x + 11$

$y = -\left(\frac{3}{2}\right)x + \left(\frac{11}{2}\right)$

Change (2) to $y = mx + b$ form.

$2y = -3x + 7$

$y = -\left(\frac{3}{2}\right)x + \left(\frac{7}{2}\right)$



The equations are inconsistent or parallel.

c. Change (1) to $y = mx + b$ form.

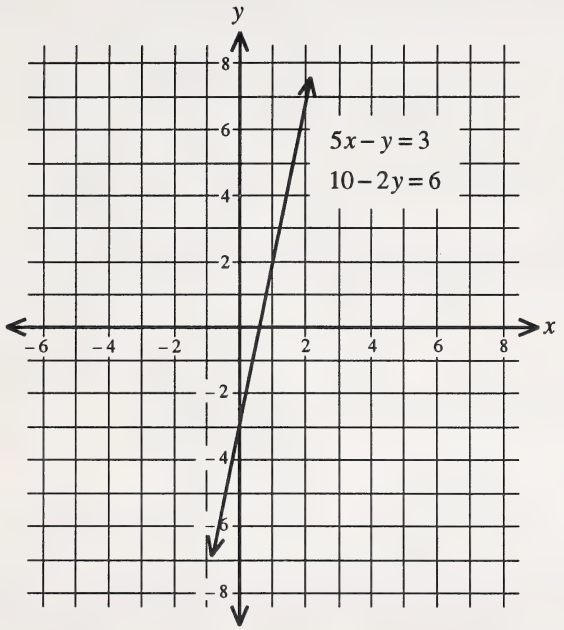
$-y = -5x + 3$

$y = 5x - 3$

Change (2) to $y = mx + b$ form.

$-2y = -10x + 6$

$y = 5x - 3$



The equations are dependent or coincident.

d. Change (1) to $y = mx + b$ form.

$-5y = -3x - 10$

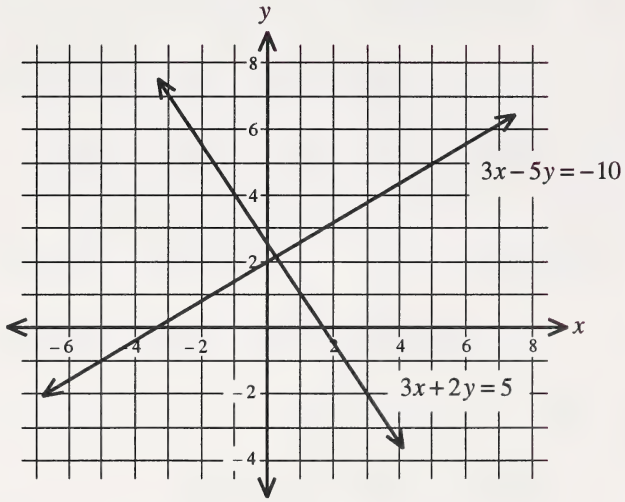
$y = \left(\frac{-3}{-5}\right)x + 2$

$y = \left(\frac{3}{5}\right)x + 2$

Change (2) to $y = mx + b$ form.

$2y = -3x + 5$

$y = -\left(\frac{3}{2}\right)x + \left(\frac{5}{2}\right)$



The equations are independent or intersecting.

Activity 2

Classify a system of two linear equations as dependent, independent, or inconsistent according to the slopes and the y-intercepts of the equations.

1. a. From (1): $5x-20y=10$ (Subtract $5x$ from both sides.)

$$-20y=-5x+10$$

$$\frac{-20y}{-20}=\frac{-5x}{-20}+\frac{10}{-20}$$ (Divide by -20.)

$$y=\frac{1}{4}x-\frac{1}{2}$$

Therefore, the slope is $\frac{1}{4}$ and the y-intercept is $-\frac{1}{2}$.

From (2): $3x-12y=6$

$$-12y=-3x+6$$

$$\frac{-12y}{-12}=\frac{-3x}{-12}+\frac{6}{-12}$$ (Divide by -12.)

$$y=\frac{1}{4}x-\frac{1}{2}$$

Therefore, the slope is $\frac{1}{4}$ and the y-intercept is $-\frac{1}{2}$.

Since the slopes and the y-intercepts are equal, the system is dependent.

b. From (1): $2x+4y=6$ (Subtract $2x$ from both sides.)

$$4y=-2x+6$$

$$\frac{4y}{4}=\frac{-2x}{4}+\frac{6}{4}$$ (Divide every term by 4.)

$$y=-\frac{1}{2}x+\frac{3}{2}$$

Therefore, the slope is $-\frac{1}{2}$ and the y-intercept is $\frac{3}{2}$.

From (2): $7x+14y=21$ (Subtract $7x$ from both sides.)

$$14y=-7x+21$$

$$\frac{14y}{14}=\frac{-7x}{14}+\frac{21}{14}$$ (Divide every term by 14.)

$$y=-\frac{1}{2}x+\frac{3}{2}$$

Therefore, the slope is $-\frac{1}{2}$ and the y-intercept is $\frac{3}{2}$.

Since both slopes and both y-intercepts are equal, the system is dependent.

Note: In a dependent system, the two lines coincide.

2. a. From (1): $7x-5y=13$

$$-5y=-7x+13$$

$$y=\frac{7}{5}x-\frac{13}{5}$$

Therefore, the slope is $\frac{7}{5}$ and the y-intercept is $-\frac{13}{5}$.

From (2): $14x-10y=7$

$$-10y=-14x+7$$

$$y=\frac{7}{5}x-\frac{7}{10}$$

Therefore, the slope is $\frac{7}{5}$ and the y-intercept is $-\frac{7}{10}$.

The slopes are equal, and the y-intercepts are different.
Therefore, the system is inconsistent.

Note: In an inconsistent system, the two lines are parallel.

b. From (1): $5x+8y=11$

$$8y=-5x+11$$

$$y=-\frac{5}{8}x+\frac{11}{8}$$

Therefore, the slope is $-\frac{5}{8}$ and the y-intercept is $\frac{11}{8}$.

From (2): $10x+16y=14$

$$16y=-10x+14$$

$$y=-\frac{5}{8}x+\frac{7}{8}$$

Therefore, the slope is $-\frac{5}{8}$ and the y-intercept is $\frac{7}{8}$.

Since the slopes are equal and the y-intercepts are different, the system is inconsistent.

3. a. From (1): $3x-8y=12$

$$-8y=-3x+12$$

$$y=\frac{3}{8}x-\frac{3}{2}$$

Therefore, the slope is $\frac{3}{8}$ and the y-intercept is $-\frac{3}{2}$.

From (2): $2x-8y=4$

$$-8y=-2x+4$$

$$y=\frac{1}{4}x-\frac{1}{2}$$

Therefore, the slope is $\frac{1}{4}$ and the y-intercept is $-\frac{1}{2}$.

Since the slopes and y-intercepts are different, the system is independent.

b. From (1): $6x-7y=15$

$$-7y=-6x+15$$

$$y=\frac{6}{7}x-\frac{15}{7}$$

Therefore, the slope is $\frac{6}{7}$ and the y-intercept is $-\frac{15}{7}$.

From (2): $3x-4y=12$

$$-4y=-3x+12$$

$$y=\frac{3}{4}x-3$$

Therefore, the slope is $\frac{3}{4}$ and the y-intercept is -3.

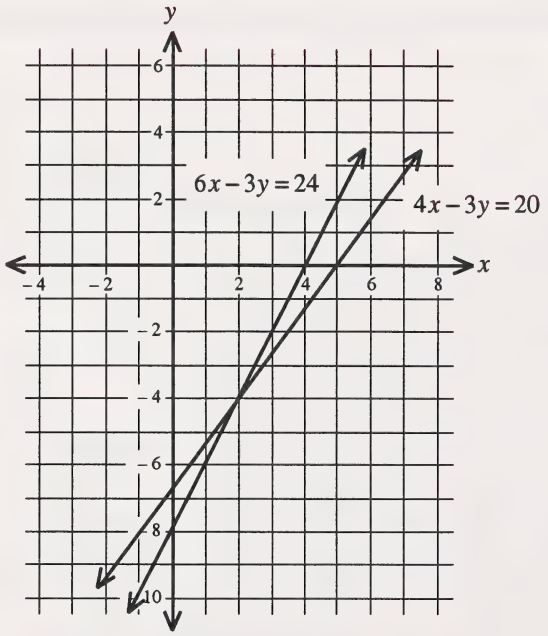
Since the slopes and y-intercepts are different, the system is independent.

Note: In an independent system, the two lines intersect.

Activity 3

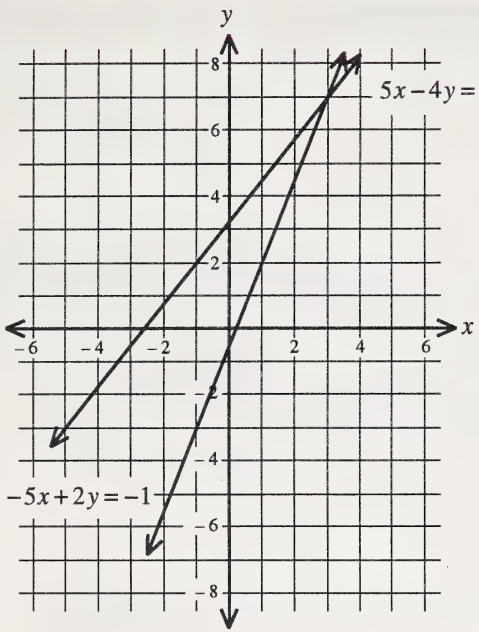
Use a graphing calculator to solve a sytem of equations.

1.



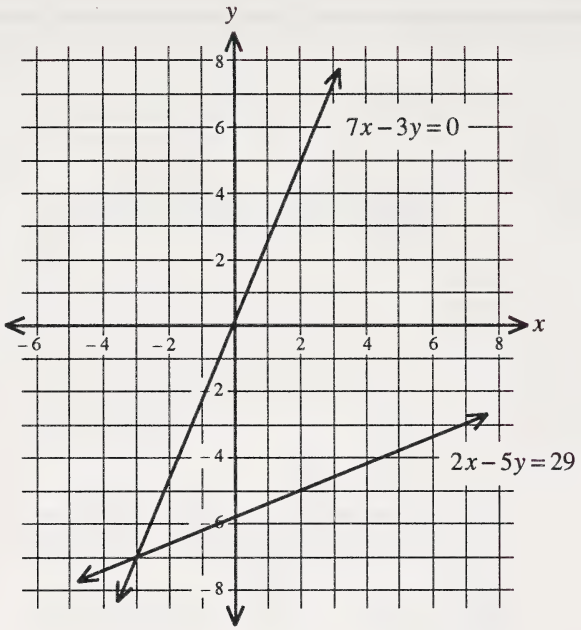
The solution is $x=2$ and $y=-4$, or the point of intersection is $(2,-4)$.

2.



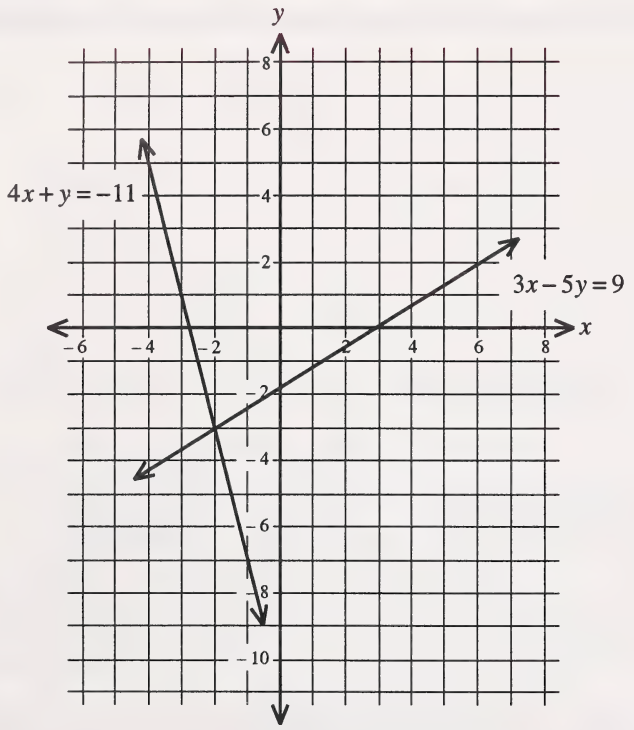
The solution is $x = 3$ and $y = 7$, or the point of intersection is $(3, 7)$.

3.



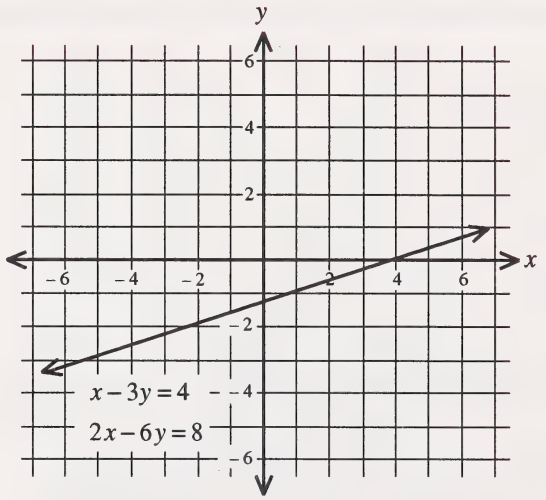
The solution is $x = -3$ and $y = -7$, or the point of intersection is $(-3, -7)$.

4.



The solution is $x = -2$ and $y = -3$, or the point of intersection is $(-2, -3)$.

5.



There are an infinite number of solutions.

Extra Help

1. Step 1: Remove $3x$ from the left side by **subtracting $3x$ from both sides**.

$$3x-5y-3x=7-3x$$
$$-5y=-3x+7$$

Step 2: Isolate y by **dividing each term by (-5)**.

$$\frac{-5y}{(-5)}=\frac{-3x}{(-5)}+\frac{7}{(-5)}$$

Step 3: Simplify the equation.

$$y=\frac{3}{5}x-\frac{7}{5}$$

$\frac{3}{5}$ is positive because the quotient of two negative numbers is positive.

2. a. $3x-11y=9$

$$-11y=-3x+9$$
$$y=\frac{-3x}{(-11)}+\frac{9}{(-11)}$$
$$y=\frac{3}{11}x-\frac{9}{11}$$

The slope is $\frac{3}{11}$ and the y-intercept is $\frac{-9}{11}$.

b. $5x+7y=19$

$$7y=-5x+19$$
$$y=\frac{-5}{7}x+\frac{19}{7}$$

The slope is $\frac{-5}{7}$ and the y-intercept is $\frac{19}{7}$.

c. $8x-9y=0$

$$-9y=-8x$$
$$y=\frac{-8}{-9}x$$
$$y=\frac{8}{9}x$$

The slope is $\frac{8}{9}$ and the y-intercept is 0.

d. $2x-12y=-3$

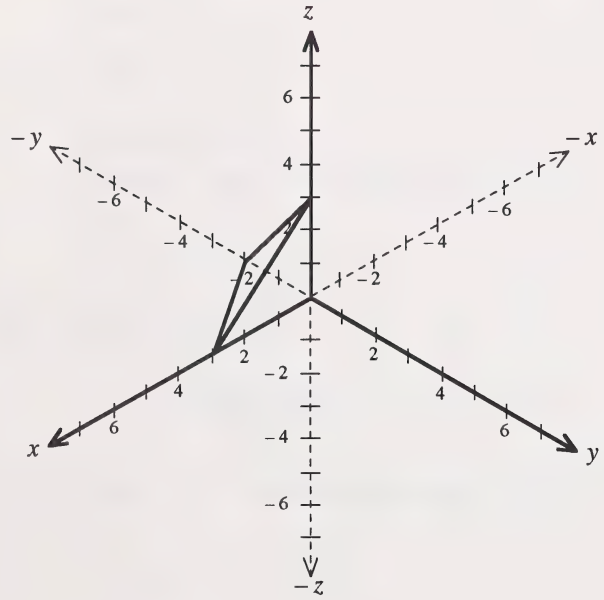
$$-12y=-2x-3$$
$$y=\frac{-2}{(-12)}x-\frac{3}{(-12)}$$
$$y=\frac{1}{6}x+\frac{1}{4}$$

The slope is $\frac{1}{6}$ and the y-intercept is $\frac{1}{4}$.

Extensions

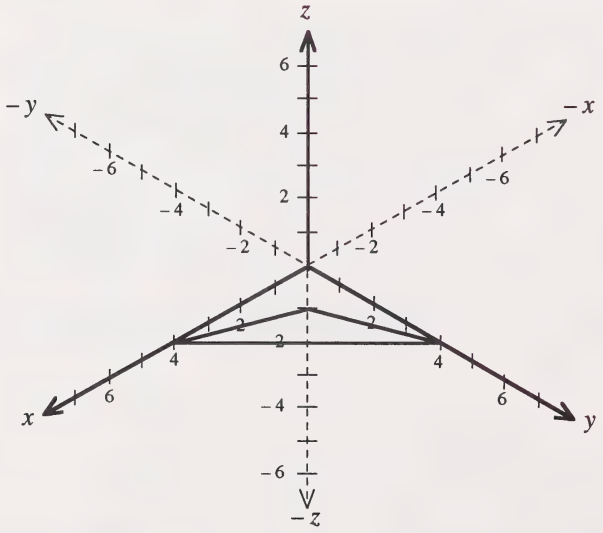
1. $2x-3y+2z=6$

   If $x=0$ and $y=0$, then $z=3$.
   If $x=0$ and $z=0$, then $y=-2$.
   If $y=0$ and $z=0$, then $x=3$.



2. $x+y-4z=4$

   If $x=0$ and $y=0$, then $z=-1$.
   If $x=0$ and $z=0$, then $y=4$.
   If $y=0$ and $z=0$, then $x=4$.



Exploring Topic 2

Activity 1

Solve a system of two linear equations by comparison.

1. a. Isolate y in each equation.

From (1): $2x - y = 4$ (Add y to both sides.)

$2x = y + 4$ (Subtract 4 from both sides.)

$2x - 4 = y$ (3)

From (2): $5x + y = 17$ (Subtract $5x$ from both sides.)

$y = -5x + 17$ (4)

Compare (3) and (4).

$2x - 4 = -5x + 17$ (Add $5x$ to both sides.)

$7x - 4 = 17$ (Add 4 to both sides.)

$7x = 21$ (Divide both sides by 7.)

$x = 3$

Substitute $x = 3$ into (1).

$2(3) - y = 4$

$6 - y = 4$

$-y = -2$

$y = 2$

The solution is $x = 3$ and $y = 2$.

b. Isolate $2x$ in each equation.

From (1): $2x + 3y = 20$ (Subtract $3y$ from both sides.)

$2x = -3y + 20$ (3)

From (2): $2x - 7y = -42$ (Add $7y$ to both sides.)

$2x = 7y - 42$ (4)

Compare (3) and (4).

$-3y + 20 = 7y - 42$

$-3y - 7y = -42 - 20$

$-10y = -62$

$y = \frac{-62}{-10}$

$= \frac{31}{5}$ or 6.2

Substitute $y = \frac{31}{5}$ into (1).

$$2x + 3\left(\frac{31}{5}\right) = 20$$

$$2x + \frac{93}{5} = 20$$

$$2x = 20 - \frac{93}{5}$$

$$2x = \frac{100}{5} - \frac{93}{5}$$

$$2x = \frac{7}{5}$$

$$x = \frac{7}{5} \times \frac{1}{2}$$

$$= \frac{7}{10} \text{ or } 0.7$$

The solution is $x = \frac{7}{10}$ and $y = \frac{31}{5}$.

2. a. From (1): $2x = -3y + 19$ (3)

From (2): $2x = \frac{1}{3}y - 1$ (4)

Compare (3) and (4).

$-3y + 19 = \frac{1}{3}y - 1$ (Multiply each term by 3.)

$-9y + 57 = y - 3$

$-9y - y = -3 - 57$

$-10y = -60$

$y = 6$

Substitute $y = 6$ into (1).

$2x + 3(6) = 19$

$2x + 18 = 19$

$2x = 1$

$x = \frac{1}{2}$

The solution is $x = \frac{1}{2}$ and $y = 6$.

b. From (1): $-5y = 1 - \frac{1}{2}x$ (3)

From (2): $-5y = 11 - 3x$ (4)

Compare (3) and (4).

$1 - \frac{1}{2}x = 11 - 3x$ (Multiply each term by 2.)

$2 - x = 22 - 6x$

$-x + 6x = 22 - 2$

$5x = 20$

$x = 4$

Substitute $x = 4$ into (2).

$3(4) - 5y = 11$

$12 - 5y = 11$

$-5y = -1$

$y = \frac{1}{5}$

The solution is $x = 4$ and $y = \frac{1}{5}$.

Activity 2

Solve a system of two linear equations by substitution.

1. a. From (1): $x = 5y - 25$ (3)

Substitute (3) into (2).

$8x + 7y = 82$

$8(5y - 25) + 7y = 82$

$40y - 200 + 7y = 82$

$47y - 200 = 82$

$47y = 282$

$y = \frac{282}{47}$

$= 6$

Substitute $y = 6$ in (3).

$x = 5(6) - 25$

$= 30 - 25$

$= 5$

The solution is $x = 5$ and $y = 6$.

b. From (2): $-y=-4x+5$ (Subtract $4x$ from both sides.)

$y=4x-5$ (3) (Divide both sides by -1.)

Substitute (3) into (1).

$$5x+2y=29$$
$$5x+2(4x-5)=29$$
$$5x+8x-10=29$$
$$13x=39$$
$$x=3$$

Substitute $x=3$ into (3).

$$y=4(3)-5$$
$$=12-5$$
$$=7$$

The solution is $x=3$ and $y=7$.

2. a. From (1): $x=-6y+6$ (3)

Substitute (3) into (2).

$\frac{1}{4}x+3y=2$ (2)

$\frac{1}{4}(-6y+6)+3y=2$ (Multiply every term by 4.)

$$-6y+6+12y=8$$
$$6y+6=8$$
$$6y=2$$
$$y=\frac{1}{3}$$

Substitute $y=\frac{1}{3}$ into (3).

$$x=-6\left(\frac{1}{3}\right)+6$$
$$=-2+6$$
$$=4$$

The solution is $x=4$ and $y=\frac{1}{3}$.

b. From (1): $-y=-3x+8\frac{4}{5}$

$y=3x-8\frac{4}{5}$ or $y=3x-\frac{44}{5}$ (3)

Substitute (3) into (2).

$\frac{1}{6}x+\frac{5}{2}y=1$

$\frac{1}{6}x+\frac{5}{2}\left(3x-\frac{44}{5}\right)=1$

$\frac{1}{6}x+\frac{15}{2}x-22=1$ (Multiply every term by 6.)

$x+45x-132=6$

$46x=138$

$x=3$

Substitute $x=3$ into (3).

$y=3x-\frac{44}{5}$

$=3(3)-\frac{44}{5}$

$=9-\frac{44}{5}$

$=\frac{45}{5}-\frac{44}{5}$

$=\frac{1}{5}$

The solution is $x=3$ and $y=\frac{1}{5}$.

Activity 3

Solve a system of two linear equations using addition or subtraction.

1. a. $2x-3y=4$ (1)

$7x-2y=31$ (2)

(1) $\times 2$: $4x-6y=8$ (3)

(2) $\times 3$: $21x-6y=93$ (4)

(4) $-$ (3): $17x+0y=85$

$17x=85$

$x=\frac{85}{17}$

$=5$

Substitute $x=5$ into (1).

$2(5)-3y=4$

$10-3y=4$

$-3y=-6$

$y=2$

The solution is $x=5$ and $y=2$.

b. $5x-2y=7$ (1)
$7x-3y=9$ (2)

(1) $\times 3$: $15x-6y=21$ (3)
(2) $\times 2$: $14x-6y=18$ (4)
(3) $-$ (4): $x-0y=3$
$x=3$

Substitute $x=3$ into (1).

$5(3)-2y=7$
$15-2y=7$
$-2y=-8$
$y=4$

The solution is $x=3$ and $y=4$.

2. a. $3x-5y=7$ (1)
$6x+7y=31$ (2)

$6x+7y=31$ (2)
(1) $\times 2$: $6x-10y=14$ (3)
(2) $-$ (3): $0x+17y=17$
$17y=17$
$y=1$

Substitute $y=1$ into (1).

$3x-5(1)=7$
$3x-5=7$
$3x=12$
$x=4$

The solution is $x=4$ and $y=1$.

b. $2x+y=16$ (1)
$3x-2y=3$ (2)

$3x-2y=3$ (2)
(1) $\times 2$: $4x+2y=32$ (3)
(2) $+$ (3): $7x+0y=35$
$7x=35$
$x=5$

Substitute $x=5$ into (1).

$2(5)+y=16$
$10+y=16$
$y=6$

The solution is $x=5$ and $y=6$.

3. a. $\frac{1}{2}x+3y=31$ (1)
$\frac{1}{4}x+\frac{1}{3}y=5$ (2)

$(1) \times 2$: $x + 6y = 62$ (3)

$(2) \times 12$: $3x + 4y = 60$ (4)

$(3) \times 3$: $3x + 18y = 186$ (5)

$(5) - (4)$: $0x + 14y = 126$

$14y = 126$

$y = \frac{126}{14}$

$= 9$

Substitute $y = 9$ into (3).

$x + 6(9) = 62$

$x + 54 = 62$

$x = 8$

The solution is $x = 8$ and $y = 9$.

b. $\frac{1}{3}x - 2y = -4$ (1)

$\frac{1}{2}x + \frac{1}{3}y = 4$ (2)

$(1) \times 3$: $x - 6y = -12$ (3)

$(2) \times 6$: $3x + 2y = 24$ (4)

$x - 6y = -12$ (3)

$(4) \times 3$: $9x + 6y = 72$ (5)

$(3) + (5)$: $10x + 0y = 60$

$10x = 60$

$x = 6$

Substitute $x = 6$ into (4).

$3(6) + 2y = 24$

$18 + 2y = 24$

$2y = 6$

$y = 3$

The solution is $x = 6$ and $y = 3$.

4. a. $0.2x + 3y = 6.3$ (1)

$0.3x + 2y = 4.45$ (2)

$(1) \times 2$: $0.4x + 6y = 12.6$ (3)

$(2) \times 3$: $0.9x + 6y = 13.35$ (4)

$(4) - (3)$: $0.5x + 0y = 0.75$

$0.5x = 0.75$

$x = \frac{0.75}{0.5}$

$= 1.5$

Substitute $x = 1.5$ into (1).

$0.2(1.5) + 3y = 6.3$

$0.3 + 3y = 6.3$

$3y = 6$

$y = 2$

The solution is $x = 1.5$ and $y = 2$.

b. $0.2x+0.7y=5$ (1)
$3x-0.5y=9$ (2)

(1) $\times 0.5$: $0.1x+0.35y=2.5$ (3)
(2) $\times 0.7$: $2.1x-0.35y=6.3$ (4)
(3) + (4): $2.2x+0y=8.8$

$$2.2x=8.8$$
$$x=\frac{8.8}{2.2}$$
$$=4$$

Substitute $x=4$ into (2).

$$3(4)-0.5y=9$$
$$12-0.5y=9$$
$$-0.5y=-3$$
$$y=6$$

The solution is $x=4$ and $y=6$.

Activity 4

Solve a system of two linear equations using any algebraic method.

1. Use the substitution method because y can be easily isolated in (1).
2. Use the substitution method because x can be easily isolated in (1).
3. Use the comparison method because both equations can be equated to $5x$.
4. Use the elimination method because the substitution and comparison methods would be too complicated.
5. Use the elimination method because the substitution and comparison methods would be too complicated.

Activity 5

Solve word problems using any algebraic method.

1. a. $x+4y=7.90$

   b. Let x be the number of dimes and y the number of quarters.
   $10x$ = value of dimes
   $25y$ = value of quarters
   $\therefore 10x+25y=600$ (change dollars to cents)

   c. $2(x-y)=(x+y)-7$

2. Let x be the smaller number and y the larger number.

   $x=y-3$ (1)
   $x+y=99$ (2)

Substitute ① into ②.

$(y-3)+y=99$

$2y-3=99$

$2y=102$

$y=51$

Substitute $y=51$ into ②.

$x=51-3$

$=48$

Therefore, the two numbers are 48 and 51.

3. Let x be the larger number and y the smaller number

$x-y=24$ ①

$2y-x=17$ ②

From ①: $x=y+24$ ③

Substitute ③ into ②.

$2y-(y+24)=17$

$2y-y-24=17$

$y-24=17$

$y=41$

Substitute $y=41$ into ③.

$x=41+24$

$=65$

Therefore, the two numbers are 65 and 41.

4. Let x be the cost of a kilogram of turkey and y the cost of a kilogram of club steak.

$3x+4y=35.20$ ①

10% more means 110% of the original price.
20% more means 120% of the original price.

$110\%(3x)+120\%(4y)=41.52$

$1.1(3x)+1.2(4y)=41.52$ ②

From ②: $3.3x+4.8y=41.52$ ③

① × 1.1: $3.3x+4.4y=38.72$ ④

③ − ④: $0x+0.4y=2.8$

$0.4y=2.8$

$y=\$7.00$

Substitute $y=7$ into ①.

$3x+4(7)=35.2$

$3x+28=35.2$

$3x=7.2$

$x=\$2.40$

Therefore, the cost of a kilogram of turkey is \$2.40, and the cost of a kilogram of club steak is \$7.00.

5. Let x be the cost of a litre of oil and y the cost of a litre of soft drink.

5% discount = 95% original price
20% discount = 80% original price

$4x+2y=\$7.60$ (1)

$0.95(4x)+0.8(2y)=\$6.80$ (2)

From (2): $3.8x+1.6y=6.80$ (3)

(1) $\times 0.95$: $3.8x+1.9y=7.22$ (4)

(4) $-$ (3): $0x+0.3y=0.42$

$0.3y=0.42$

$y=1.4$

Substitute $y=1.4$ into (1).

$4x+2(1.4)=7.6$

$4x+2.8=7.6$

$4x=4.8$

$x=1.2$

The cost of a litre of oil is \$1.20, and the cost of a litre of a soft drink is \$1.40.

6. Let x be the amount invested at 7% and y the amount invested at 11%.

$0.07x+0.11y=235$ (1)

$x+y=2500$ (2)

From (2): $x=2500-y$ (3)

Substitute (3) into (1).

$0.07(2500-y)+0.11(y)=235$

$175-0.07y+0.11y=235$

$175+0.04y=235$

$0.04y=60$

$y=\dfrac{60}{0.04}$

$=\$1500$

Substitute $y=1500$ in (2).

$x=2500-1500$

$=1000$

Whitley invested \$1500 at 11% and \$1000 at 7%.

7. Let x be the amount invested at 12% and y the amount invested at 9%.

$x+y=3000$ (1)

$0.12x-0.09y=150$ (2)

From (1): $x=3000-y$ (3)

Substitute (3) into (2).

$$0.12(3000-y)-0.09y=150$$
$$360-0.12y-0.09y=150$$
$$360-0.21y=150$$
$$-0.21y=-210$$
$$y=\frac{-210}{-0.21}$$
$$=\$1000$$

Substitute $y=1000$ into (3).

$x=3000-1000$

$=\$2000$

Denzil invested \$2000 at 12% and \$1000 at 9%.

8. Let x be the units digit and the y the tens digit.

$x+y=7$ (1)

$3x=1+2y$ (2)

From (1): $x=7-y$ (3)

Substitute (3) into (2).

$$3(7-y)=1+2y$$
$$21-3y=1+2y$$
$$21-1=2y+3y$$
$$5y=20$$
$$y=4$$

Substitute $y=4$ into (1).

$x=7-4$

$=3$

Therefore, the number is 43.

9. Let x be the units digit and the y the tens digit.

$y=x+5$ (1)

$10y+x=10(x+y)$ (2)

The two digit number is $10y+x$. For example, if the number is 34, x (units digit) would be 4 and y (tens digit) would be 3, but the number is $10y+x$ since $10(3)+4=34$.

Substitute ① into ②.

$$10(x+5)+x=10(x+x+5)$$
$$10x+50+x=10(2x+5)$$
$$11x+50=20x+50$$
$$11x-20x=50-50$$
$$-9x=0$$
$$x=0$$

Substitute $x=0$ into ①.

$$y=0+5$$
$$=5$$

Therefore, the two digit number is 50.

10. Let x be the speed of the boat in still water and y the speed of the current.

Upstream: speed of the boat $=x-y$
Downstream: speed of the boat $=x+y$
Distance = time × speed

$$(3\text{ h})\times(x-y)=9\text{ km} \quad ①$$
$$(2\text{ h})\times(x+y)=9\text{ km} \quad ②$$

From ①: $3(x-y)=9$

$$3x-3y=9 \quad ③$$

From ②: $2(x+y)=9$

$$2x+2y=9 \quad ④$$

③ × 2: $6x-6y=18$ ⑤
④ × 3: $6x+6y=27$ ⑥
⑤ + ⑥: $12x+0y=45$

$$12x=45$$
$$x=\frac{45}{12}$$
$$x=3.75$$

Substitute $x=3.75$ into ③.

$$3(3.75)-3y=9$$
$$11.25-3y=9$$
$$-3y=9-11.25$$
$$-3y=-2.25$$
$$y=0.75$$

Therefore, the speed of the boat in still water is 3.75 km/h and the speed of the current is 0.75 km/h.

11. Let x be the distance Peter drove at 100 km/h and y the distance Peter drove at 110 km/h.

The time travelled at 100 km/h is $\frac{x}{100}$.

The time travelled at 110 km/h is $\frac{y}{110}$.

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$x + y = 295 \quad (1)$$

$$\frac{x}{100} + \frac{y}{110} = 2\frac{3}{4} \quad (2)$$

From (1): $y = 295 - x$ (3)

Substitute (3) into (2).

$$\frac{x}{100} + \frac{295 - x}{110} = 2\frac{3}{4}$$

(Multiply every term by 1100 because 1100 is the L.C.M of 100, 110, and 4.)

$$11x + 2950 - 10x = 3025$$
$$x + 2950 = 3025$$
$$x = 75$$

Peter drove 75 km at 100 km/h.

Extra Help

1. Change the percentages to decimal numbers

Step 1: 10% is equal to **0.10**.

12% is equal to **0.12**.

Step 2: The interest earned at 10% is $x \times$ **0.10**.

The interest earned at 12% is $y \times$ **0.12**.

Step 3: Therefore, the total interest is

$0.10x + 0.12y = 640$. (1)

Therefore, the total investment is $x + y = 6000$. (2)

Step 4: Since (1) has decimal coefficients, multiply (1) by 100 so that all the decimal coefficients can be changed to whole number coefficients.

(1) $\times 100$: **10x** + **12y** $= 64\,000$ (3)

Step 5: In order to make the coefficient of x in (2) and (3) equal, you have to multiply (2) by 10.

$$10x + 12y = 64\,000 \quad (3)$$
(2) $\times 10$: $10x + 10y = 60\,000$ (4)
(3) $-$ (4): $0x + 2y = 4000$
$$2y = 4000$$
$$y = 2000$$

Step 6: Substitute $y = 2000$ into (2).

$x + 2000 = 6000$

$x =$ **6000** $-$ **2000**

$=$ **4000**

The investment at 10% is **$4000** and the investment at 12% is **$2000**.

2. Let x be the investment at 6% and y the investment at 8%.

$x + y = 1500$ (1)

$0.06x + 0.08y = 110$ (2)

From (1): $x = 1500 - y$ (3)

Substitute (3) into (2).

$0.06(1500 - y) + 0.08y = 110$

$90 - 0.06 + 0.08y = 110$

$90 + 0.02y = 110$

$0.02y = 20$

$y = \frac{20}{0.02}$

$= 1000$

Substitute $y = 1000$ into (3).

$x = 1500 - 1000$

$= 500$

Karen invested $500 at 6% and $1000 at 8%.

Extensions

1. $2x - 3y = 7$ (1)

$4x - 6y = 8$ (2)

(1) $\times 2$: $4x - 6y = 14$ (3)

$4x - 6y = 8$ (2)

(3) $-$ (2): $0 = 6$

Since $0 \neq 6$, there is no solution. Therefore, the system is inconsistent.

2. $3x - 5y = 6$ (1)

$6x - 10y = 12$ (2)

(1) $\times 2$: $6x - 10y = 12$ (3)

$6x - 10y = 12$ (2)

(3) $-$ (2): $0 = 0$ (4)

This indicates that there are an infinite number of solutions therefore, the system is dependent.

3. $x-y+z=7$ (1)
$2x+y-3z=5$ (2)
$x-2y+z=10$ (3)

(1) + (2): $3x-2z=12$ (4) (Eliminates y.)
$2\times$(2) + (3): $5x-5z=20$ (5) (Eliminates y.)
(4) $\times 5$: $15x-10z=60$ (6)
(5) $\times 2$: $10x-10z=40$ (7)
(6) − (7): $5x+0z=20$
$5x=20$
$x=\frac{20}{5}$
$=4$

Substitute $x=4$ into (4).

$3(4)-2z=12$
$12-2z=12$
$-2z=0$
$z=0$

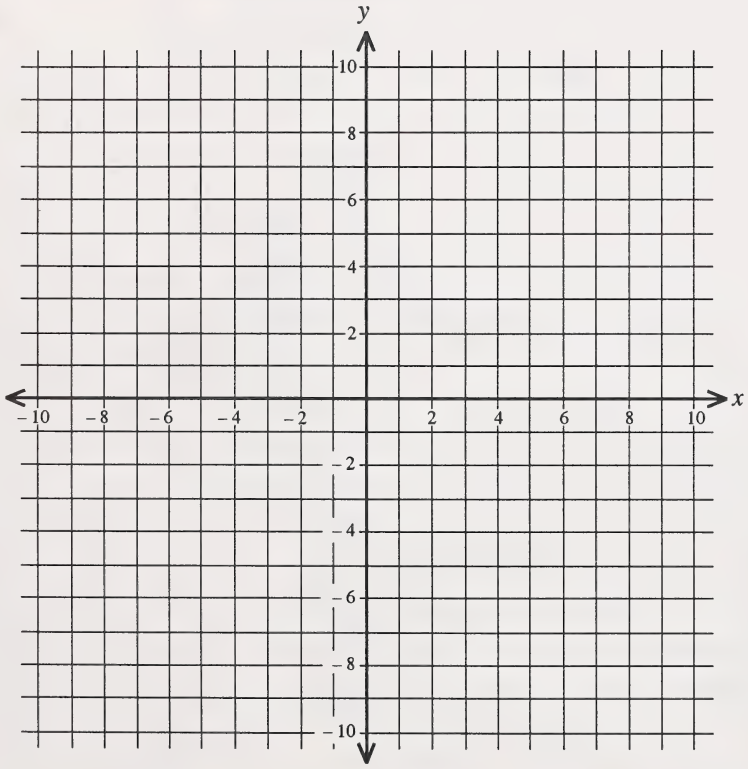
Substitute $x=4$ and $z=0$ into (1).

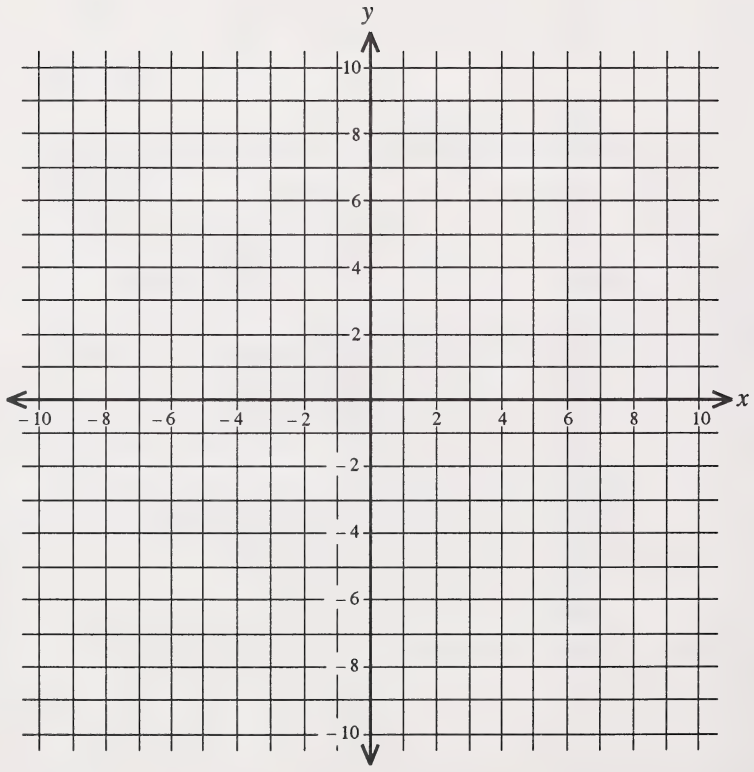
$4-y+0=7$
$-y=3$
$y=-3$

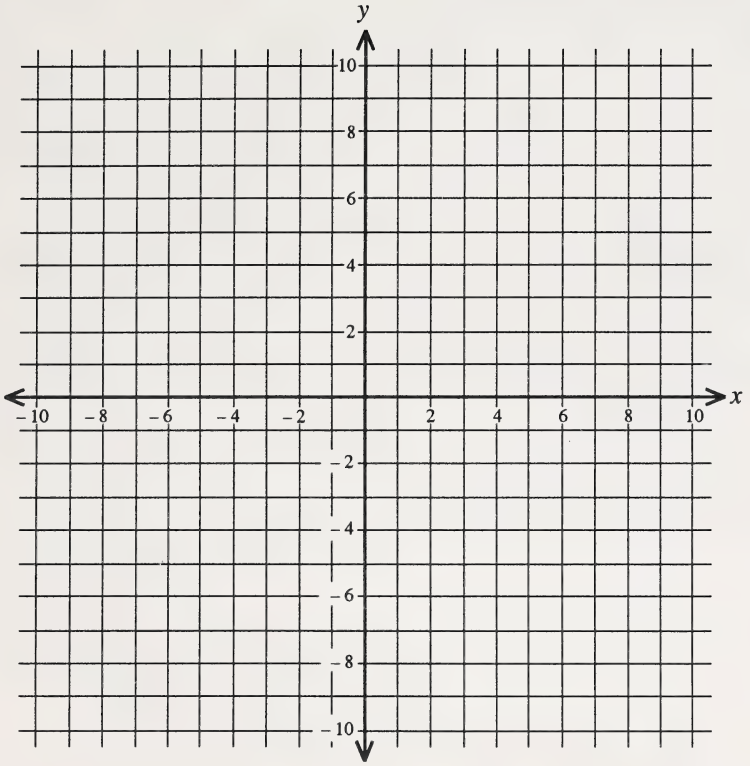
The solution is $x=4$, $y=-3$, and $z=0$.

Appendix B
Grids

Grids



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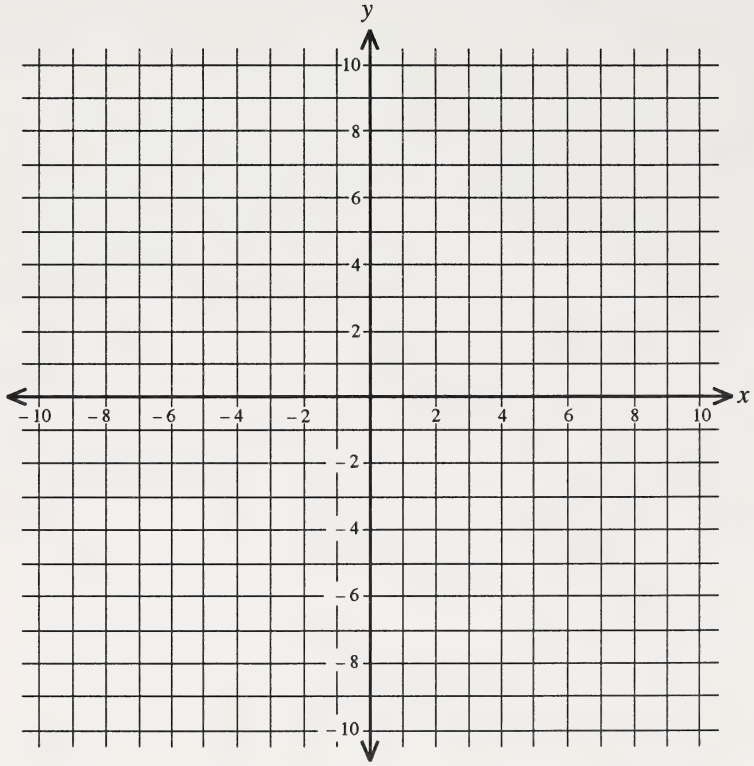
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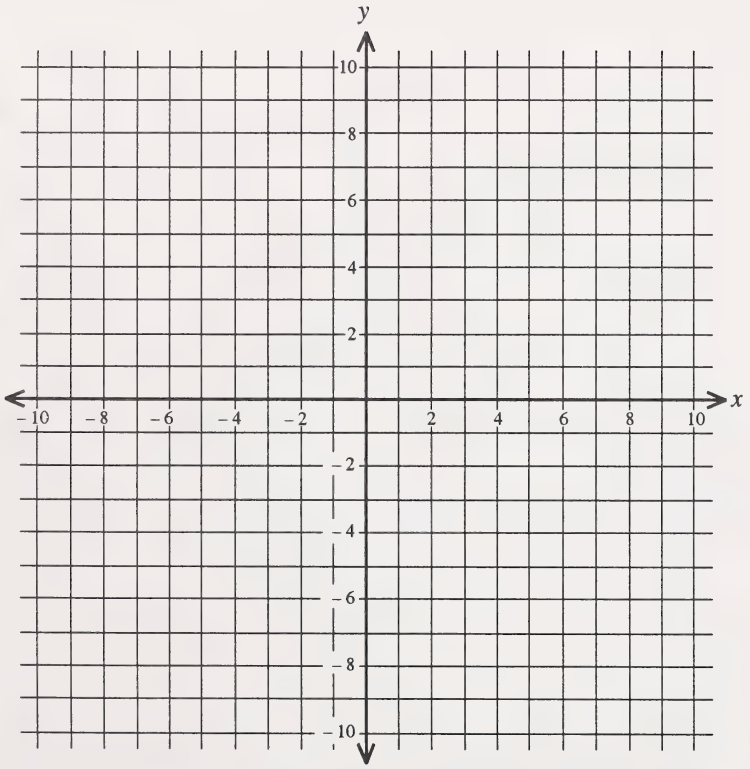
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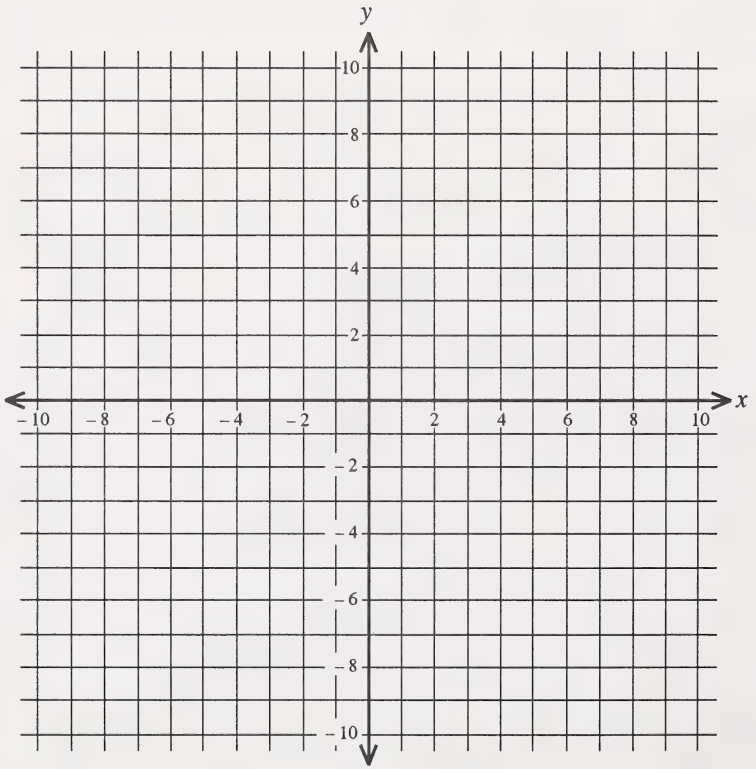
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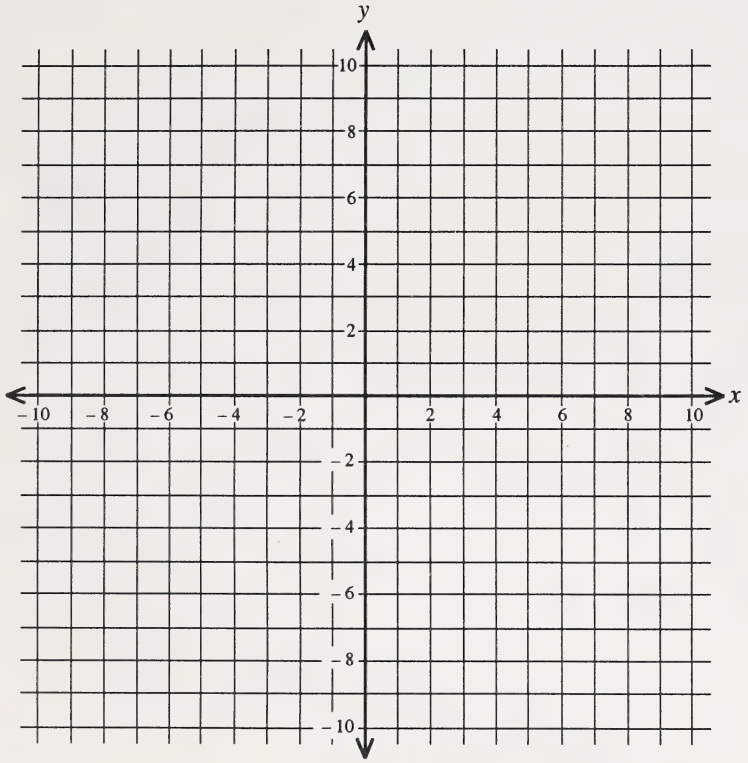
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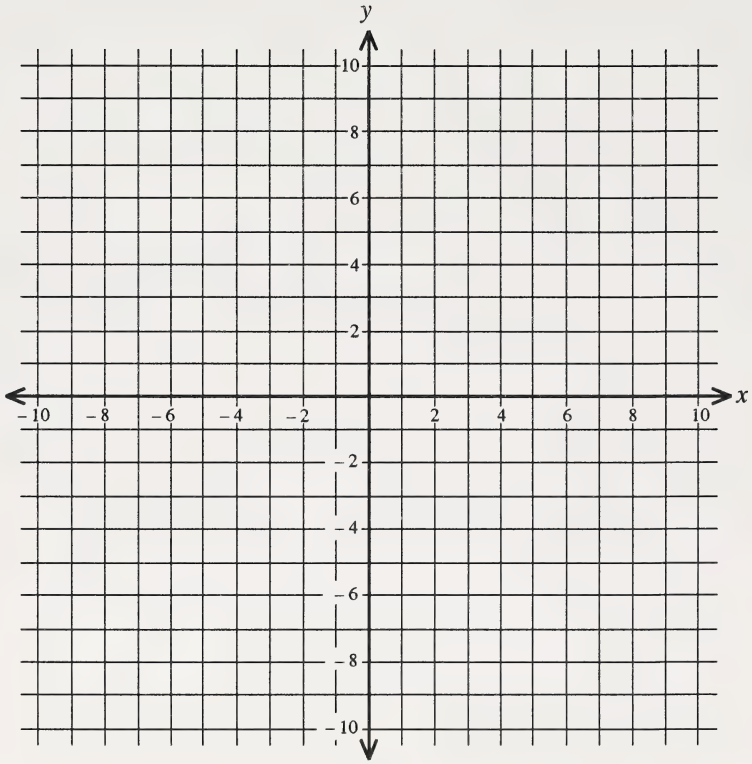
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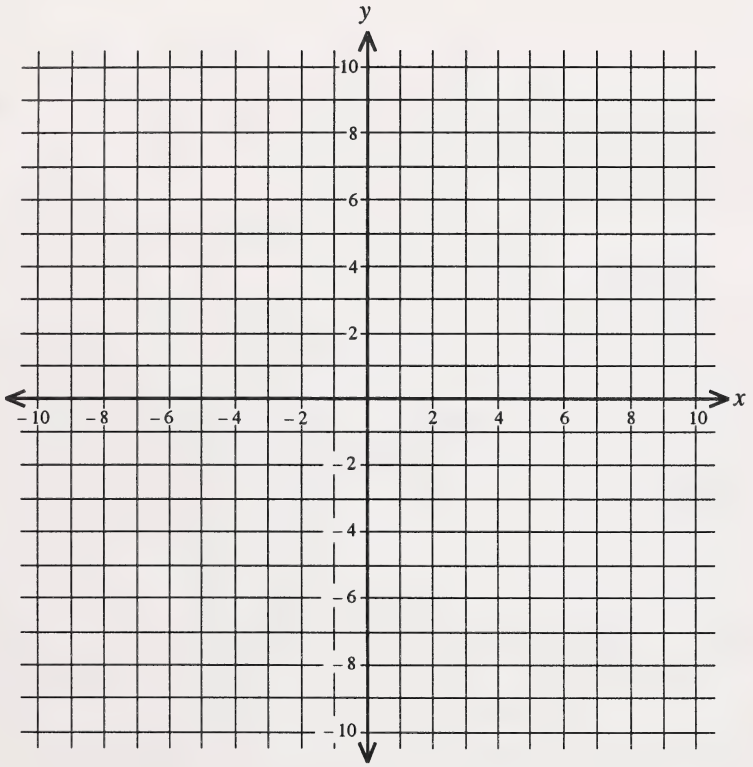
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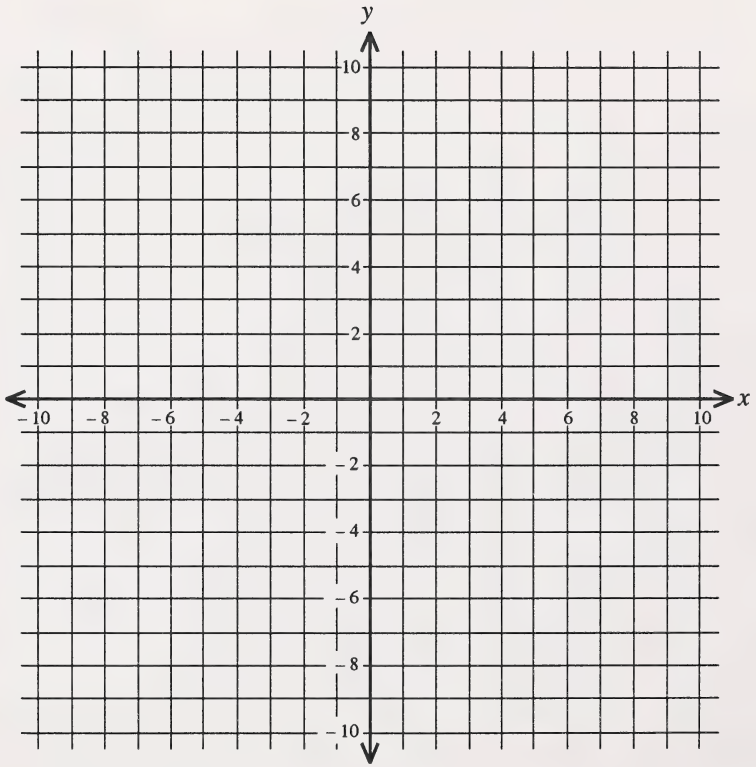
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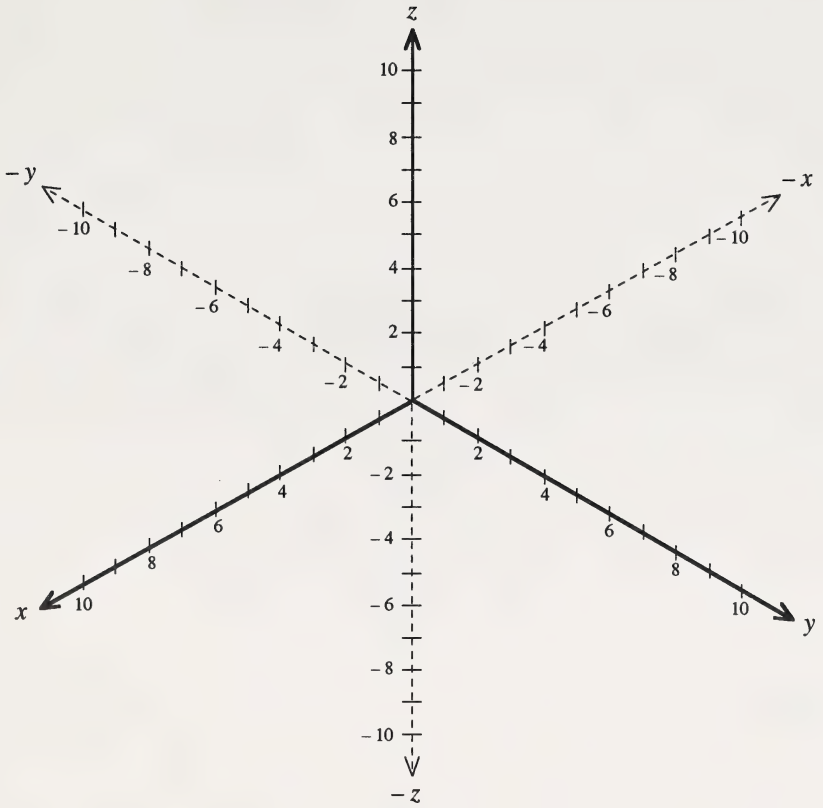
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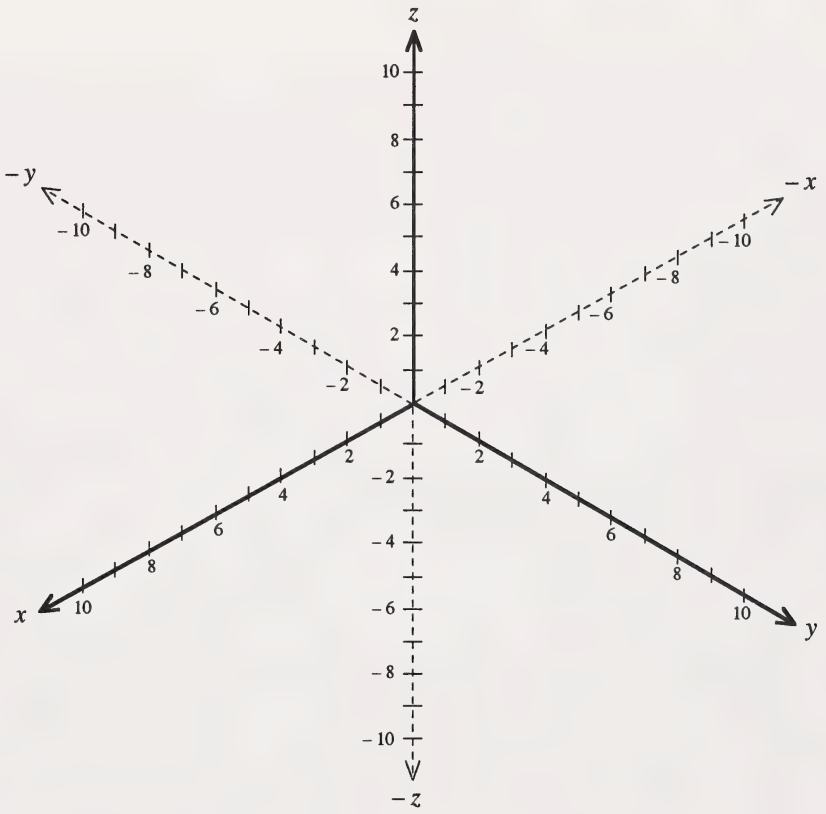
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L.R.D.C.
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Mathematics 10

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Unit 6

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